Chapter 4
Processes in Micro-Fracture Continuum

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Abstract In the frame of the asymmetric continuum theory we present some aspects of the micro-fracture processes. An extension of this theory, accommodating for a significant increase of external load, can describe some features of a progressively granulated and fractured material. According to our theory, two kinds of motions – displacements and rotations – in some source processes, e.g., in an earthquake source, may be generated independently or with some phase shift due to the rebound release processes. The asymmetric theory includes a possible phase shift between the simultaneous solutions for the displacement and rotation motions. Equivalently, we present the solutions with the simultaneous strain and rotation motions or those with the $\pi/2$ phase shift between them. Such specific solutions can explain the synchronization action in fracture processes.

Further, we describe on this basis some properties of the slip motion along a fracture zone and micro-fragmentation appearing in the flattened vortex process. Our approach relates to the processes under a very high confining load. The derived nonlinear equations are discussed.

4.1 Introduction

We present some elements of the asymmetric continuum theory with important applications; our consideration on the asymmetric continuum theory includes:

- balance laws for the symmetric and antisymmetric stresses and related wave fields,
- hypothesis of a synchronization process based on the rebound processes and the wave solution with the phase shift between the strains and rotations,
- formation of a granulated (mylonite) zone during fracture processes.

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4.2 Asymmetric Continuum

One version of the asymmetric theories of elasticity has been founded by Nowacki (1986); this theory includes the couple-stresses introduced similarly as in the micropolar and micromorphic theories (see: Eringen, 1999). Our version of the asymmetric theory includes the asymmetric stresses, symmetric strains and anti-symmetric rotations and permits for a possible phase shift between the displacement and rotation motions. For the constitutive laws joining the antisymmetric stresses and rotations we follow some ideas introduced by Shimbo (1975; 1995) and related considerations on the friction processes and rotation of grains.

Experimental evidences for the appearance of rotation and shear oscillation (twist) in a seismic field are based on the records of seismic rotation fields (see: Teisseyre et al.(eds), 2006; Teisseyre K.P., 2007).

4.2.1 Standard asymmetric continuum

We follow the asymmetric theory with the asymmetric stresses, S_{kl}, and deformations, symmetric strains - E_{kl} and rotations - ω_{kl} (Teisseyre, 2009; Teisseyre, Chapter 3, this issue):

$$S_{kl} = S_{(kl)} + S_{[kl]}, \quad E_{kl} = E_{(kl)}, \quad \omega_{kl} = \omega_{[kl]} \tag{4.1}$$

Let us underline that the energy stored, E, becomes related also to rotational deformation:

$$E = \frac{1}{2} S_{kl}(E_{kl} + \omega_{kl}) = \frac{1}{2} S_{(kl)} E_{kl} + \frac{1}{2} S_{[kl]} \omega_{kl}$$

Instead of the Kröner method (Kröner, 1981) based on the self-fields we introduce the material structure indexes, e^0 and χ^0, joining the deformation fields, strains and rotations, with the displacement motions:

$$\begin{aligned} E_{kl} &= e^0 E^0_{kl} = e^0 \frac{1}{2}\left(\frac{\partial}{\partial x_k} u_l + \frac{\partial}{\partial x_l} u_k\right), \\ \omega_{kl} &= \chi^0 \omega^0_{kl} = \chi^0 \left[\frac{\partial}{\partial x_k} u_l - \frac{\partial}{\partial x_l} u_k\right]. \end{aligned} \tag{4.2a}$$

For $\chi^0 = 0$, $e^0 = 1$ we return to classic elasticity, while for $e^0 = 0$, $\chi^0 = 1$, we would have a continuum with the rigid, densely packed spheres with friction subjected to an external moment load.

The independent fields (E_{kl}, ω_{kl}) lead us to defects and extreme deformations (Teisseyre and Gorski, 2009).

For solid elastic bodies we will put simply:

$$e^0 = 1, \chi^0; \qquad E_{kl} = E^0_{kl}, \qquad \omega_{kl} = \chi^0 \omega^0_{kl}, \tag{4.2b}$$

where the phase index χ^0 may vary from 0 to $\chi^0 = \{\pm 1, \pm \mathrm{i}\}$

The Shimbo consideration (1975) supplements the classical constitutive relation:

$$S_{(kl)} = \lambda \delta_{kl} E_{ss} + 2\mu E_{kl}, \qquad S_{[kl]} = 2\mu\omega_{kl} \tag{4.3}$$

We will consider the simplified system of motion equations at a constant density (Teisseyre, 2009);

$$(\lambda + 2\mu)\frac{\partial^2}{\partial x_k \partial x_k} E_{ss} = \rho \frac{\partial^2}{\partial t^2} E_{ss} - \frac{\partial^2}{\partial x_s \partial x_s} p \tag{4.4a}$$

$$\mu \frac{\partial^2 E^D_{nl}}{\partial x_k \partial x_k} - \rho \frac{\partial^2 E^D_{nl}}{\partial t^2} = -(\lambda + \mu)\left(\frac{\partial^2 E_{ss}}{\partial x_l \partial x_n} - \frac{\delta_{nl}}{3}\frac{\partial^2 E_{ss}}{\partial x_k \partial x_k}\right) + Y_{nl} \tag{4.4b}$$

where

$$Y_{nl} = \frac{1}{2}\left(\frac{\partial F_n}{\partial x_l} + \frac{\partial F_l}{\partial x_n} - \frac{\partial^2 p}{\partial x_n \partial x_l} + \frac{1}{3}\frac{\partial^2}{\partial x_n \partial x_l} p\right) \quad \text{at} \quad \frac{\partial F_n}{\partial x_n} = 0 \;\text{ and }\; \partial u_k / \partial x_k = 0$$

The field, E^D_{nl}, can be used to define the shear-twist vector meaning the rotational oscillations of the shear axes and its amplitude (cf., Teisseyre, 2009).

For the independent rotation we write, instead of the balance of the angular moments

$$\mu \Delta \omega_{ki} = \rho \frac{\partial^2 \omega_{ki}}{\partial t^2} + K_{[ki]} \tag{4.5a}$$

where the balance of stress moments is replaced with that for the antisymmetric stresses:

$$\frac{\partial}{\partial x_k} M_{pk} = \varepsilon_{pki} l^2 \frac{\partial^2}{\partial x_k \partial x_n} S_{[ni]} = 2\mu \varepsilon_{pki} l^2 \frac{\partial^2 \omega_{ki}}{\partial x_n \partial x_n} = 2\mu \varepsilon_{pki} l^2 \frac{\partial^2 \omega_{ki}}{\partial x_n \partial x_n} \tag{4.5b}$$

Here, we shall also note the important equivalence

$$\varepsilon_{pki} \frac{\partial^2 \omega_{ni}}{\partial x_k \partial x_n} = \varepsilon_{pki} \frac{\partial^2 \omega_{ki}}{\partial x_n \partial x_n} \;\text{ at the condition }\; \varepsilon_{pki} \frac{\partial \omega_{ki}}{\partial x_p} = 0, \quad \text{or} \quad \frac{\partial \omega_p}{\partial x_p} = 0 \tag{4.5c}$$

We can write that the force moment relates to the angular moment; this statement leads to a definition employing an effective rotation motion, Ω:

$$\frac{\partial}{\partial x_k} M_{pk} = 2\mu\varepsilon_{pki} l^2 \frac{\partial^2 \omega_{ni}}{\partial x_k \partial x_n} \rightarrow M_{pk} = 2\mu\varepsilon_{pki} l^2 \frac{\partial \omega_{ni}}{\partial x_n} = 2\mu\varepsilon_{pki} l \Omega_i, \quad \Omega_i = l \frac{\partial \omega_{ni}}{\partial x_n} \tag{4.6a}$$

including the neighboring rotating elements with the adequately defined characteristic Cosserat length. Instead of (4.5a) we write :

$$\mu \frac{\partial^2 \Omega_i}{\partial x_n \partial x_n} = \rho \frac{\partial^2 \Omega_i}{\partial t^2} + K_i \tag{4.6b}$$

4.3 Slip and fragmentation transport in fracture micro-continuum

In a solid continuum, the advanced deformations - slip and fragmentation processes - could be described with the help of the Navier-Stokes transport idea; we may explain such processes with the help of the slip-transport, υ, and fragmentation-transport $\tilde{\upsilon}$.

Considering a simple case with a constant density, we can transform the displacement motion equation exchanging the partial time derivatives to the total ones:

$$\begin{aligned} &\frac{\partial}{\partial t} u_i \rightarrow \frac{\mathrm{d}}{\mathrm{d}t} u_i = \frac{\partial}{\partial t} u_i + \upsilon_s \frac{\partial}{\partial x_s} u_i \quad \text{and} \\ &\frac{\partial^2}{\partial t^2} u_i \rightarrow \frac{\mathrm{d}^2}{\mathrm{d}t^2} u_i = \left(\frac{\partial}{\partial t} + \upsilon_s \frac{\partial}{\partial x_s}\right)\left(\frac{\partial}{\partial t} u_i + \upsilon_s \frac{\partial}{\partial x_s} u_i\right) \end{aligned} \tag{4.7}$$

and we arrive at a type of the Navier-Stokes transport equation related to slip in solids:

$$\rho \frac{\mathrm{d}^2}{\mathrm{d}t^2} u_i = \rho \frac{\mathrm{d}}{\mathrm{d}t} \upsilon_i = \tilde{\eta} \frac{\partial^2}{\partial x_k \partial x_k} \upsilon_i - F_i$$

or

$$\frac{\partial^2 u_i}{\partial t^2} + \frac{\partial \upsilon_k}{\partial t} \frac{\partial u_i}{\partial x_k} + 2\upsilon_k \frac{\partial \upsilon_i}{\partial x_k} + \upsilon_k \frac{\partial \upsilon_s}{\partial x_k} \frac{\partial u_i}{\partial x_s} + \upsilon_k \upsilon_s \frac{\partial^2}{\partial x_k \partial x_s} u_i = \frac{\mu}{\rho} \frac{\partial^2 u_i}{\partial x_k \partial x_k} - \frac{F_i}{\rho}, \tag{4.8}$$

where in the particular case we may put $\upsilon_i = \frac{\partial}{\partial t} u_i$.

The obtained relation (4.8) describes transport process related to a slip micro-fracture process.

For the fragmentation phenomena and fragmentation/transport we can introduce the arcuate transport process. Thus, when in equation (4.6b) we consider the rotation transport with a related characteristic length, l, serving as a rotation arm:

$$\tilde{\upsilon}_k = \varepsilon_{ksn} l_s \frac{\partial \Omega_n}{\partial t}, \quad \frac{\mathrm{d}}{\mathrm{d}t} \to \frac{\partial}{\partial t} + \tilde{\upsilon}_s \frac{\partial}{\partial x_s}, \tag{4.9}$$

then we obtain (in a constant density case):

$$\frac{\partial^2 \Omega_{ni}}{\partial t^2} + \frac{\partial \tilde{\upsilon}_k}{\partial t} \frac{\partial \Omega_{ni}}{\partial x_k} + 2\tilde{\upsilon}_k \frac{\partial \Omega_{ni}}{\partial x_k} + \tilde{\upsilon}_k \frac{\partial \tilde{\upsilon}_s}{\partial x_k} \frac{\partial \Omega_{ni}}{\partial x_s} + \tilde{\upsilon}_k \tilde{\upsilon}_s \frac{\partial^2 \Omega_{ni}}{\partial x_k \partial x_s} = \frac{\mu}{\rho} \Delta \Omega_{ni} - \frac{K_{[ni]}}{\rho}. \tag{4.10}$$

Further, we focus on the vortex motions with the vortices oriented along the z-axis. On the plane $z = \text{const}$ we may have some variable characteristic length, L, related to a possible vorticity, while along the z-axis the characteristic length will remain very small:

$$l_k = \{L, L, l\}; \quad L \gg l \tag{4.11}$$

We pass to the cylindrical coordinate system; the transport (4.9) becomes as follows:

$$\tilde{\upsilon}_r \approx L \frac{\partial \Omega_z}{\partial t}, \quad \tilde{\upsilon}_\varphi \approx -L \frac{\partial \Omega_z}{\partial t}, \quad \tilde{\upsilon}_z \approx L \left(\frac{\partial \Omega_\varphi}{\partial t} - \frac{\partial \Omega_r}{\partial t} \right) \tag{4.12}$$

Accordingly, the total time derivative becomes

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + L \left\{ \frac{\partial \Omega_z}{\partial t} \left(\frac{\partial}{\partial r} - \frac{\partial}{r \partial \varphi} \right) + \left(\frac{\partial \Omega_\varphi}{\partial t} - \frac{\partial \Omega_r}{\partial t} \right) \frac{\partial}{\partial z} \right\} \tag{4.13a}$$

and for $\Omega_r \ll \Omega_z, \;\; \Omega_\varphi \ll \Omega_z \quad \text{and} \quad L(r, \varphi, z)$

$$\begin{aligned}
\frac{\mathrm{d}^2}{\mathrm{d}t^2} &= \left\{ \frac{\partial}{\partial t} + L \frac{\partial \Omega_z}{\partial t} \left(\frac{\partial}{\partial r} - \frac{\partial}{r \partial \varphi} \right) \right\} \left\{ \frac{\partial}{\partial t} + L \frac{\partial \Omega_z}{\partial t} \left(\frac{\partial}{\partial r} - \frac{\partial}{r \partial \varphi} \right) \right\} \\
&= \frac{\partial^2}{\partial t^2} + L^2 \left(\frac{\partial \Omega_z}{\partial t} \right)^2 \left(\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{r^2 \partial \varphi^2} - 2 \frac{\partial^2}{r \partial r \partial \varphi} \right) + 2L \frac{\partial \Omega_z}{\partial t} \left(\frac{\partial^2}{r \partial t \partial \varphi} - \frac{\partial^2}{\partial t \partial r} \right) \\
&\quad + L \frac{\partial^2 \Omega_z}{\partial t^2} \left(\frac{\partial}{\partial r} - \frac{\partial}{r \partial \varphi} \right) + L^2 \left(\frac{\partial \Omega_z}{\partial t} \right)^2 \left(\frac{\partial \Omega_z}{\partial r} - \frac{\partial \Omega_z}{r \partial \varphi} \right) \left(\frac{\partial}{\partial r} - \frac{\partial}{r \partial \varphi} \right)
\end{aligned} \tag{4.13b}$$

Thus, for the macroscopic rotation field (4.6b) (external forces omitted) we obtain:

$$\frac{\tilde{\mu}}{\tilde{\rho}}\Delta\Omega_z - \frac{\partial^2\Omega_z}{\partial t^2} - M_z = 0 \tag{4.14a}$$

where the constants $\tilde{\mu}$ and $\tilde{\rho}$ relate to a medium with the advanced micro-fracture, and we have put the transport term:

$$M_z = 2L^2\left(\frac{\partial\Omega_z}{\partial t}\right)^2\left(\frac{\partial^2\Omega_z}{\partial r^2} + \frac{\partial^2\Omega_z}{r^2\partial\varphi^2} - 2\frac{\partial^2\Omega_z}{r\partial r\partial\varphi}\right) + 2L\frac{\partial\Omega_z}{\partial t}\left(\frac{\partial^2\Omega_z}{\partial t\partial r} - \frac{\partial^2\Omega_z}{r\partial t\partial\varphi}\right) + L\frac{\partial^2\Omega_z}{\partial t^2}\left(\frac{\partial\Omega_z}{\partial r} - \frac{\partial\Omega_z}{r\partial\varphi}\right) \tag{4.14b}$$

The obtained relation describes the overall transport processes in the micro-fracture medium.

4.4 Local transport in sources of asymmetric elastic continuum

Maintaining the motion equations (4.4) and (4.6) we introduce into the source definition the transport term, M, as defined in (4.14b); this form of a local transport is based on hidden micro-transport elements related to a local slip or fragmentation. Here, we will consider a problem in which the micro-fracture processes concentrated in a source concern a fragmentation (rotation effects) and can be expressed by a source rotation moment introduced into equation (4.6); we put

$$M_z(\Omega_z, L) \to K_z, \quad \text{and} \quad K_z = K\exp(-\alpha t); \quad K = M = \text{constant} \tag{4.15a}$$

where Ω_z follows from the solution (4.6) and the condition that K remains constant.

As mentioned above, the condition for K (to be constant), includes also some material degradation effects and we have assumed that these local degradation processes can be related to the transport phenomena concentrated in a source.

According to (4.15a) we put

$$M_z = M\exp(-\alpha t), \quad \Omega_z = \Omega\exp(-\alpha t), \tag{4.15b}$$

and for the variable vortex radius $L(r,z)$, using (4.14b), we obtain the condition:

$$2L^2\alpha^2\Omega^2\frac{\partial^2\Omega}{\partial r^2} + 3\alpha^2 L\Omega\frac{\partial\Omega}{\partial r} - \alpha^2\Omega = M \tag{4.15c}$$

The solution for Ω shall be found from (4.6) or (4.14a) for M constant; under a plane shear load rotation field, in solids, it shall depend on φ, however, we can

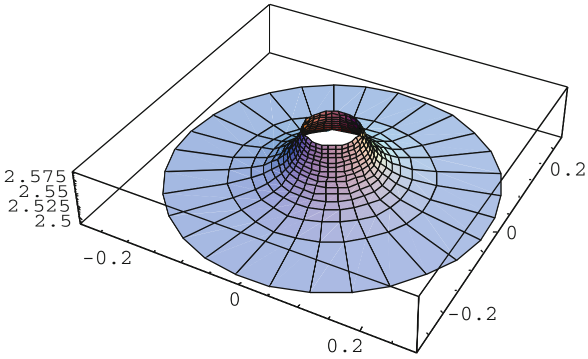

Fig. 4.1 Vortex structure concentrated in the vicinity of the source fragmentation plane

express this dependance through the function sin2φ as follows from the consecutive angular changes of the shears along a plane. We obtain from (4.15b):

$$\frac{\partial^2\Omega}{\partial r^2}+\frac{\partial\Omega}{r\partial r}+\frac{\partial^2\Omega}{r^2\partial\varphi^2}+\frac{\partial^2\Omega}{\partial z^2}-\alpha^2\tilde{\rho}\Omega=\frac{\partial^2\Omega}{\partial r^2}+\frac{\partial\Omega}{r\partial r}-\frac{4\Omega}{r^2}+\frac{\partial^2\Omega}{\partial z^2}-\alpha^2\tilde{\rho}\Omega=\frac{1}{\tilde{\mu}}M \tag{4.16}$$

The respective numerical solution of this equation may be used to compute from (4.15c) the fragmentation arm changes, L, which enables us to present this fragmentation similarly to a vortex structure (Fig. 4.1). The presented changes of this vortex arm can be revealed when computing L from (4.15c) with the initial condition $L = r_0$. As presented in this figure, this numerical solution is given with a change of the scales along the vortex plane in comparison to the z-axis; we expressed this by a change of the rigidity parameter μ to $\tilde{\mu}$, as presented in eq. (4.6). In this way, we take into account the fact that, due to fragmentation, the material parameters along this plane undergo essential changes due to these micro-fracture processes.

We shall underline that the vortex process starts due to a point source, that is, reversely than it is usually considered in the vortex problems. The vortex arm increases up from a point source $r = 0$, and the vortex is concentrated near the fragmentation plane (this is quite different in comparison to vortices in fluids). This follows from our assumption that the material properties in a fragmentation plane have become changed, in comparison to those along the direction perpendicular to it, while the material properties remain almost unchanged.

4.5 Shear and confining loads

In the local micro-fracture zones, the shear and confining loads lead to the transport processes related to the displacement and rotation motions.

The transport relations for the displacement motions are especially important for the slip processes under a shear load; in that case, the rotational transport can sometimes be neglected (Fig. 4.2).

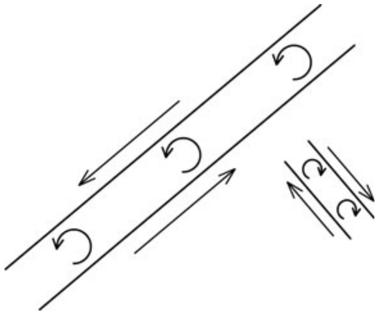

Fig. 4.2 Shear load: sketch of slip elements and the opposite rotations as appear along the main slip and a double couple partner

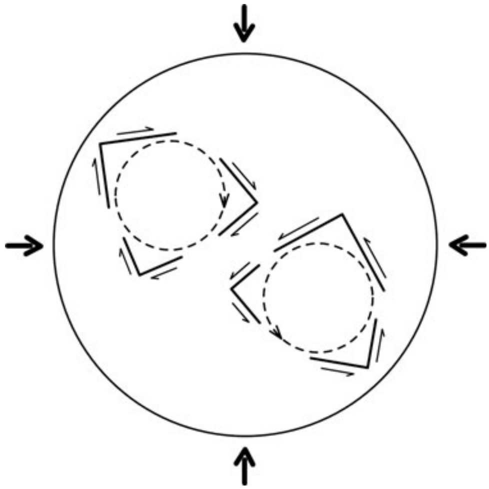

Fig. 4.3 Confining load: the induced opposite shears inside the fragmentation elements; the opposite shear couples result in a rotation of fragmented elements

Under a confining load, the fragmentation processes may be described by a helical transport incorporated into the rotation field equations. The induced shear micro-fractures appear in this case along the perpendicular slip fragments; the related induced shears would be mutually compensated inside some micro-region in which a common rotation sense will produce a micro-fragmentation circular structure (Fig. 4.3). At the neighboring fragments, the rotation sense might be opposite. However, the induced consecutive shears inside a particular fragmentation element will be manifested by the perpendicular micro-displacement couples; these rotation couples, contrary to shear couples, result in a rotational micro-fragment, while the resulting shear field will become compensated almost to null. Thus, a fracture running according to this scenario, due to induced rotation couples, leads to the material fragmentation and rotation; the directions of rotations can be opposite inside the material under an applied confining load. This is an opposite case to that of shear process, which leads to the shear nuclei, described by the double couples in the conventional meaning.

However, we shall notice that variable inner shear fields, caused by the inner micro-fractures and related stress releases at the neighboring sites, may cause the rotational oscillations of the main axes of these double couples; this kind of rotation motion is called shear-twist (Teisseyre, 2009).

However, to get a better understanding of the fracture processes we may consider jointly the micro-slip and micro-fragmentation processes. When approaching the micro-fracture and fracture states we should consider also the consecutive substantial changes in the material properties and in the governing equations. The material properties undergo changes, e.g., from the elastic to plastic and, further on, to crushed, granulated and even partly melted mylonite.

Moreover, an influence of the rotation processes of various nature and scale may be of great importance when some vortex micro-structures will appear. To outline such an approach we may follow the asymmetric continuum theory; we start with the relations presented above (Eqs. 4.1–4.5) as concerns the perfect elasticity. During a further deterioration, related to plastic flow and micro-fracture processes, we assume that the compressibility relation, expressed by means of the axial part of stresses and strains, remains practically unchanged (however, some changes in the value of compressibility can be easily included).

Under a shear load, the micro-fracture processes can proceed as follows: shear stresses and related strains cause some changes in the angular molecule orientations, then the slip motion and break of the molecular bonds start with an immediate drop of shears, and then there appears the rebound rotation retarded in phase. Under a compression load, the induced defects cause an appearance of the opposite shear centers, then some micro-breaks lead to the rotations and fragmentation process, and then there appear the rebound slip motions retarded in phase. In the first case, the shears create the dynamic angular deformations leading to the bond breaks and slip propagation followed by the rebound rotations retarded in phase. In the second case, the micro-fractures under compression lead to the opposite sense of the induced shear motions: the twist motions and the related fragmentation and granulation processes precede the slip rebounds retarded in phase.

4.6 Conclusions

The considered rotational (helical) transport processes are expected to occur in the granulated structures or those undergoing the micro-fragmentation processes; the presented new development in solid theory with the micro-fracture and fragmentation processes describes the independent rebound release processes occurring with a possible phase shift between the rotation and shear-twist oscillations. Any torque moment caused by the independent transport and micro-fracture may generate the spin and shear-twist motions.

The slip-fracture and fragmentation processes caused by a joint action of shear and confining loads may run with a mutual interaction. The equations for the joint slip-shear and fragmentation-pressure processes take into account a possible shift between the related oscillations and the rebound release dynamics.

The obtained relations for the helical transport differ essentially from those for fluids; here, we deal with the square time rates of the transport contributions; this is related to the transition: $\partial^2/\partial t^2 \rightarrow \mathrm{d}^2/\mathrm{d}t^2$ and to the material changes due to fragmentation process.

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