Chapter 1
Nonlinear Dynamics as a Tool for Revealing Synchronization and Ordering in Geophysical Time Series: Application to Caucasus Seismicity

Teimuraz Matcharashvili and Tamaz Chelidze

1.1 Introduction

It is a common statement in scientific literature that the complexity of nature has always been an inevitable problem in our efforts towards understanding spatial forms of natural objects and temporal evolution of natural processes. "Complex" and "complexity" are now quite popular scientific terms, though there is little consensus on their official definitions and they still have a variety of meanings depending on the context [Arecci, 1996; Shiner, 1999]. This is so because the study of complexity in both dynamical and structural sense is in its infancy, being at the same time a rapidly developing field in the forefront of many areas of science, including mathematics, physics, geophysics, economics, biology, etc.

Natural systems and/or processes are complex mainly due to their nonlinearity, an intrinsic property of the underlying laws conditioning the absence of determinism of the Universe. The presence of this property is revealed in the specificity of systems whose temporal behavior and spatial structures were named "complex" [Kantz, 1997; Matcharashvili, 2000]. In order to avoid misunderstanding caused by the tradition associating the term nonlinearity exclusively with dynamics, it should be stressed that at present the terms nonlinearity and complexity are commonly regarded as synonyms. This is convenient in order to address both complex nonlinear temporal evolution and complex non-Euclidean spatial forms of natural systems. As an inherent property, nonlinearity or complexity is revealed in the absence of deterministic cause-effect relation observed on different spatial and temporal scales. This property incorporates phenomena with a very broad diversity

T. Matcharashvili (✉)
Georgian Technical University, 77 Kostava ave., 0171 Tbilisi, Georgia
M. Nodia Institute of Geophysics, 1 Alexidze str., 0171 Tbilisi, Georgia
e-mail: matcharashvili@gtu.ge

T. Chelidze
M. Nodia Institute of Geophysics, 1 Alexidze str., 0171 Tbilisi, Georgia
e-mail: tamaz.chelidze@gmail.com

V. de Rubeis et al. (eds.), *Synchronization and Triggering: from Fracture to Earthquake Processes*, GeoPlanet: Earth and Planetary Sciences 1,
DOI 10.1007/978-3-642-12300-9_1, © Springer-Verlag Berlin Heidelberg 2010

of dynamical features. Generally speaking, this diversity manifests itself in a certain kind of hierarchy of dynamical behavior, ranging from strict determinism to total randomness. The most important is the fact that between these extremes there are many intermediate states that reveal different degree of orderliness, such as, e.g., periodicity, quasiperiodicity, deterministic chaos, low and high dimensional dynamics, hyperchaos, etc. [Theiler, 1997; Kantz, 1997].

Until recently, neither a qualitative detection nor a quantitative evaluation of these intermediate states has been possible because of the absence of a corresponding mathematical formalism and appropriate data analysis methods. At present, the time series nonlinear analysis universal technique has been elaborated [Packard et al, 1980; Berge et al, 1984; Eckmann et al, 1987; Abarbanel et al, 1993; Rapp et al, 1993; Kantz, Shreiber 1997], which often (but not always) enables us to achieve correct qualitative and quantitative assessment of complex processes by their dynamical characteristics.

It is necessary to mention that traditional linear methods are mostly not suitable for complex processes of interest. This is why in different fields of science and practice there has been an explosion of papers searching for methods aiming at detection of peculiarities of complex systems evolution in order to achieve reliable identification of processes by their dynamics. As the complex systems are characterized by different transitions between regular, laminar, and chaotic behaviors, the knowledge of these transitions is necessary for understanding the process. In this respect, one of the fundamental problems is how to measure the complexity of both local and global dynamical behaviors from the observed time series.

There are several main approaches to quantify the complexity of processes by analyzing the measured time series [Boffetta, 2001]. Some of them have roots in dynamical systems and fractal theory and include Lyapunov exponents, Kolmogorov-Sinai entropy, and fractal dimensions [Eckmann et al. 1987]. These methods are based on reconstruction and testing of phase space objects equivalent to the unknown dynamics. The other methods stem from the information theory including Shannon entropy [Shannon, 1948], algorithmic complexity [Shiner, 1999; Yao, 2004] etc., and are mostly based on symbolic dynamics.

For different complex systems, various approaches to complexity measurements can be used. The common problem of many methods is the requirement of long, high quality stationary data sets, which is not always easy to fulfill in analyses of real natural or laboratory systems. To overcome these difficulties, new tests have been proposed, such as recurrence plots (RP) and recurrence quantitative analysis (RQA). These methods equip us for gaining new understanding on the complex natural dynamics.

1.2 Overview of nonlinear data analysis methods

Most nonlinear data analysis methods are based on reconstruction and inspection of the state or phase space of the investigated process. When the system of interest is nonrandom, it has a property known as recurrence [Ruelle, 1994]. This means that after some transients, the system comes back close to the same points in phase space again and again. The character of time evolution of trajectory forms a phase space structure or attractor of the system. The shape of attractor provides essential information on dynamical features of the investigated process. Generally, a point in a phase space is associated with a single state of the system which is fully defined by a set of m dynamical variables. It is clear that to have a complete description of the state of the dynamic system, these m physical quantities should all be measured, at least in principle. Unfortunately, in most of experimental situations, not all (and often only a single) physical quantities of state variable can be measured; all what we have is an one-dimensional time series and from this series we have to learn as much as possible about the system that generated the signal. According to Takens' theorem it is possible to catch the essential dynamical properties of a system by a reconstruction of its phase space by only one variable. Two- and three-dimensional phase portraits encapsulating essential dynamical properties of the analyzed complex process are used as qualitative tests of the process dynamics. They enable to accomplish first qualitative visual inspection of unknown dynamics and uncover general properties of the analyzed process. Qualitative analysis allows us to reveal possible existence of specific attractors, e.g., strange ones which point to the deterministic chaotic behavior.

Further, the phase space can be analyzed using quantitative methods.

For both qualitative and quantitative approaches, the phase space should be reconstructed from measured (or simulated) data sets. Generally, the measurements commonly result in discrete time series $g_i(t)$, where $t = i\Delta t$, and Δt is the sampling rate. As a rule, the sampling rate is constant, forming equidistant time series but this is not always the case. The time series taken at time intervals of different length, the so-called unevenly sampled time series, are also quite common [Schreiber, 1999]. As far as system variables are coupled, a single component contains essential information about the dynamics of the whole system [Rapp et al., 1993; Castro, 1997; Kantz, 1997]. Therefore, the trajectory reconstructed from this scalar time series is expected to have the same properties as the trajectory embedded in the original phase space, formed by all m state variables. Packard et al. (1980) and Takens (1981) independently proposed the idea of using single sequence of measurements to transform process dynamics into the phase space structure to gain information on the unknown underlying dynamics from this structure. According to the embedding theorem, there exists a one-to-one image of attractor in the embedding space, if the embedding dimension is sufficiently high [Hegger, 1999]. The idea was successfully realized after Takens proved that it is possible to reconstruct from a single scalar time series a new attractor which is diffeomorphically equivalent to the attractor in the original state space of the system under study. Essentially two methods of reconstructions are available: delay coordinates and derivative coordinates. Derivative coordinates were originally proposed by Packard et al. (1980) and consist of using the higher order derivatives of the measured time series as the independent coordinates. Since derivatives are more susceptible to noise, this is usually not very practical for real data which are very noisy themselves. Therefore, the method of delay coordinates was recognized as a more practical tool. Delaying data by T helps to exclude distortions of analysed dynamics caused by temporal closeness of observations. The T value should be large enough to avoid insubstantial functional dependence between data and not so large to make them completely independent statistically. If these conditions are fulfilled, a set of d-dimensional vectors in d-dimensional space can be reconstructed:

$$\bar{X}(i) = [x(i), x(i+T),\ x(i+2T), \ldots, x(n+(d-1)T)]. \tag{1.1}$$

According to Takens' theorem, the reconstructed dynamics is equivalent to the dynamics of the real underlying system [Packard, 1980; Takens, 1981]. Equivalence of two dynamics means that their dynamical invariants (e.g., generalized dimensions, the Lyapunov spectrum, recurrence characteristics, etc., to be shortly described below) are identical. The delay time, T, for the reconstructions can be calculated from the autocorrelation function or mutual information (MI) first minimum. The averaged mutual information evaluates the amount of bits of information shared between two data sets over a range of time delays is defined as [Abarbanel,1993; Kantz, 1997; Cover, 1991; Kraskov, 2004]:

$$I(X,Y) = \sum_{ij}^{N} p(i,j) \log_2 \frac{p(i,j)}{p_x(i)p_y(j)}, \tag{1.2}$$

where $p_x(i)$ and $p_y(j)$ are the probabilities of finding $x(i)$ and $x(i+T)$ measurements in time series, respectively, $p(i,j)$ is a joint probability of finding measurements $x(i)$ and $x(i+T)$ in time series, and T is the time lag. It is important to mention that in contrast to the linear correlation coefficient (which also can be used for delay time calculation), MI is sensitive also to dependences which are not linear, i.e., do not manifest themselves in the covariance. MI is zero if and only if the two random variables are strictly independent. The MI calculation is also important as a tool to provide information on phase space points probability distribution.

In order to define the correct value of embedding dimension $d_e \geq 2d_a + 1$ (where d_e is the dimension of embedding space and d_a is attractor's dimension) one may use the so-called false nearest neighbor method [Kennel, 1992; Hegger, 1999]. The percentage of false nearest neighbors (phase points projected into neighborhoods of points to which they would not belong in higher dimensions) approaches zero as the dimension of the phase space increases.

Since phase space structure attractor or image of dynamics is formed, the two most popular ways for the quantitative evaluation of complexity of analyzed dynamics are: quantification of the average evolution patterns of neighboring trajectories in the state space, and/or quantification of the geometric patterns of the state space object.

Evolution of phase space trajectories could be analyzed by calculation of spectrum of Lyapunov exponents or, as it is often done, by calculation of maximal Lyapunov exponent $\lambda_{\max}$. Generally, Lyapunov exponents quantify the average exponential rate of divergence of neighbouring trajectories in the state space, and thus provide a measure of the system's response to local perturbations [Rosenstein, 1993; Kantz, 1997]. For measured data sets, the maximum Lyapunov exponent $\lambda_{\max}$ for a dynamical system can be determined from the equation: $d(t) = d_0 e^{\lambda_{\max} t}$, where $d(t)$ is the mean divergence between neighboring trajectories in the state space at time t and d_0 is the initial separation between neighboring points. There are several methods [Wolf, 1985; Sato, 1987; Rosenstein, 1993] for estimating $\lambda_{\max}$ which often suffer from drawbacks that are serious for practical use, namely, the estimates of $\lambda_{\max}$ are unreliable for small data sets and need essential computational resources. Generally, if $\lambda<0$, phase trajectories are drawing together and the considered dynamical system has an attractor in the form of a fixed point. When $\lambda=0$, the system tends to a stable limit cycle. $\lambda>0$ means that phase trajectories are moving away and such a system may be chaotic or random (Rosenstein, 1993).

In order to characterize the unknown dynamics by the geometry of their reconstructed phase structures, an algorithm for calculation of fractal dimensions of phase space point sets should be used. It is known that the fractal dimension of an attractor roughly characterizes the complexity and gives a lower bound for the number of equations or variables needed for modeling the underlying dynamical process. There are several such measures based on quantification of self-similar properties of phase space objects. These measures are: the information dimension (d_i), the Hausdorf dimension d_H, etc. [Abarbanel, 1993; Kantz, 1997]. We shortly describe here only the GPA method of computing correlation dimension or fractal dimension as proposed by Grassberger and Procaccia [1983]. In spite of difficulties in using it for real data sets, GPA remains to be the most popular and often used method for quantifying geometrical features of phase space objects. This is probably due to the simplicity of the algorithm [Bhattacharya, 1999] and the fact that the same intermediate calculations are used to estimate both dimension and entropy. The correlation sum, $C(r, N)$, quantifies the way in which the density of points in the state space scales with the size of the volume containing those points. This approach is based on the idea of correlation sum. Correlation sum $C(r)$ of set of points in the vector space is defined as the fraction of all possible pairs of points which are closer than a given distance r. The basic formula useful for practical application is

$$C(r,N) = \frac{2}{(N-w)(N-w+1)} \sum_{i=1}^{N} \sum_{j=i+w}^{N} \Theta\left(r - \left\| x_i - x_j \right\|\right), \tag{1.3}$$

where $\Theta(x)$ is the Heaviside step function, $\Theta(x)=0$ if $x \leq 0$ and $\Theta(x)=1$ if $x \geq 0$. $\left\| x_i - x_j \right\|$ is the Euclidian norm. Points with $i = j$ are excluded. w is the Theiler's window for fractal systems for time series that are long enough. For small r, $C(r) \propto r^{\nu}$ relationship is correct. Commonly, such a dependence is correct only for the restricted range of r values, the so-called scaling region. Correlation dimension ν or d_2 is defined as

$$\nu = d_2 = \lim_{r \to 0} \frac{\log C(r)}{\log(r)}. \tag{1.4}$$

In practice, the d_2 value is found from the slopes of $\log C(r, N)$ versus $\log r$ curves for different phase space dimensions. In order to achieve appropriate linearity of the relationship between $\log[C(r, N)]$ and $\log r$, one has to determine local slopes, or the so-called ''local scaling exponents''[Kantz, 1997]. The true correlation dimension of an unknown process is the saturation value of d_2, which does not change by increasing phase space dimension. If saturation does not take place, the correlation dimension is infinitely large which is typical for random processes.

For a correct analysis it is necessary to have data sequences that are long enough, at least $N \geq 10^{d/2}$, where N is a length of time series and d is the dimension of attractor [Abarbanel, 1993]. The three dimensions mentioned above are related by $d_2 < d_i < d_H$, with equality when the points in the state space are distributed uniformly over the attractor. In spite of popularity of d_2 calculation method, findings by GPA must be interpreted with great care as it is well known that linear stochastic processes can also mimic low-dimensional dynamics [Theiler et al. 1992; Rapp et al., 1993]. In other words, the saturation of a correlation dimension and the existence of positive Lyapunov exponents cannot always be considered as a proof of deterministic chaos, predictable in sense of patterns, which is closest to quasi-periodic dynamical regime [Rapp et al. 1993; Kantz and Shreiber, 1997]. Since linear correlations lead to many spurious conclusions in nonlinear time series analyses, it is important that the obtained results be verified using the so-called surrogate data approach. This is a method to test the null hypothesis that the analyzed time series are generated by a specific process with the known linear properties [Theiler et al., 1992]. It should be stressed again that the above phase space measures have strict restrictions in the sense of time series length and are mostly relevant for low dimensional or deterministically chaotic systems. When the dynamics of the investigated process is more complex or when dimension of underlying attractor is moderately large, say $d_2 > 5$, all the results of dimensional analysis on finite amount of real data series are not grounded well enough [Schreiber, 1999]. Moreover the real data series are often very noisy, containing measurement noise as well as dynamical noise (noise interacting with dynamics), and then the conventional estimates fail as well. Therefore, when we deal with complex dynamics, a less ambitious and more realistic goal commonly applied is to search for the inherent nonlinearity of the processes, or to rank them by the extent of nonlinearity. The practical importance of this statement becomes clear in the light of known facts that in most cases the dynamical behavior of natural scale-invariant processes is non random, revealing nonlinear structure, while valid evidences of deterministic chaotic type of dynamics are very seldom [Theiler, 1997; Marzocchi et al., 1997; Goltz, 1997].

The above-mentioned method of surrogate data equips us for testing the nonlinear structure of complex dynamics (Theiler, 1992). The surrogate data is inherently a stochastic signal which mimics certain statistical properties, such as temporal autocorrelation or Fourier power spectra of the original signal. The surrogates can be constructed from the original time series on the basis of different null hypotheses. The three types of most often used surrogates address the three main hypotheses: temporally independent noise, linearly filtered noise, and nonlinear transformation of linear filtered noise. So whenever we try to quantify the degree of nonlinearity, the results of calculation of the above measures should be compared with the similar quantities for surrogate data sets. Phase randomized surrogate sets (obtained by destroying the nonlinear structure through randomization of the phases of a Fourier transform of the original time series and following invert transformation) are often used to test the null hypothesis that the time series are linearly correlated with Gaussian noise [Theiler et al., 1992]. Also a Gaussian scaled random phase (GSRP) surrogate set can be generated to address the null hypothesis that the original time series is a linearly correlated noise that has been transformed by a static, monotone nonlinearity [Rapp et al., 1993, 1994]. The GSRP surrogates are generated in a three-step procedure. At first, a Gaussian set of random numbers is generated, which has the same rank structure as the original time series. After this, the phase randomized surrogates of these Gaussian sets are constructed. Finally, the rank structure of original time series must be reordered according to the rank structure of the phase randomized Gaussian set [Theiler, 1992].

Generally, these two methods of generation of surrogates are based on shuffling of the original data set but, in the case of Gaussian scaled random phase surrogates, the controlled shuffles [Rapp et al., 1994] can give more precise and reliable results than the unstructured shuffles of the random phase surrogates.

Commonly, for testing the null hypothesis, d_2 is used as the discriminating metric. There are several ways to measure the difference between the discriminating metric measure of the original (given by M_{orig}) and the surrogate (given by M_{surr}) time series.

The most commonly used measure of the significance of the difference between the original time series and the surrogate data is given by the criterion: $S = |\langle M_{surr}\rangle - M_{orig}|/\sigma_{surr}$, where σ_{surr} denotes standard deviation of M_{surr}. The details of the procedure, as well as an analytic expression for ΔS, the uncertainty in S, are described in Theiler et al. [1992].

Alternatively, the Monte Carlo probability can be used, defined as:

$$P_M = (\text{number of cases } M \leq M_{orig})/(\text{number of cases})$$

where P_M is an empirical measure of the probability that a value of M_{surr} will be less than M_{orig}. It is particularly appropriate when the number of surrogates is small, or when the distribution of values of M obtained with surrogates is non-Gaussian (Rapp et al. 1994).

For rejecting the null hypothesis, the Barnard and Hope nonparametric test can be used (Rapp et al. 1994). With this criterion, the null hypothesis is rejected at a confidence level $p_c=1/(N_{surr}+1)$, if $M_{orig}<M_{surr}$ for all the surrogates.

One of the serious problems in real data analyses is the influence of noises. It is preferable to use the so-called nonlinear noise reduction (which in fact is phase space nonlinear filtering) instead of common linear filtering procedures. The latter, as it is well known, may destroy the original nonlinear structure of analyzed complex processes [Hegger and Kantz 1999; Schreiber, 2000]. Nonlinear noise reduction relies on the exploration of reconstructed phase space of the considered dynamical process instead of frequency information of linear filters [Hegger and Kantz, 1999; Schreiber, 1993; Kantz and Schreiber, 1997].

As it was many times pointed out above, most methods of analysis need rather long and stationary data sets, which is commonly not typical of the measured time series. This was a strong impetus for a further development of new techniques to get an insight into the complex processes, having not very long and rather noisy observable time series. For this purpose, several measures of complexity, mostly based on a symbolic dynamics approach, have been proposed, such as Renyi entropies, the effective measure complexity, the ε and Lempel-Ziv complexity (LZC) measure, etc. [Lempel, 1976; Wackerbauer, 1994; Rapp, 2001]. The LZC is especially suitable for relatively short real data sets because is not so demanding as to the time series length as other methods [Zhang, 1999; Matcharashvili, 2001].

It is necessary to mention the approach based on the study of attractor's organization, or testing of topology of phase space images of unknown dynamics. This technique, oriented on exploration of phase space structure or image of dynamics, is the method of recurrence plots (RP) [Eckmann et al., 1987]. Let us recall here that if the dynamical system has any deterministic structure, an attractor appears in the state space. As it was already mentioned, the attractor is a set of points in phase space, towards which a dynamical path will converge. Again, the recurrence is a fundamental property of nonrandom dynamical systems, the state of which, although exponentially diverges under small disturbances, but after some time the system will come back to a state that is arbitrarily close to a former state. Recurrence plots visualize such a recurrent behavior of dynamical system. Real processes are usually characterized by complex dynamics to be embedded in high-dimensional phase spaces. RP enables to investigate structure in these high-dimensional phase spaces through a two-dimensional representation of its recurrences. It is most important to say that the recurrence plot method is effective for nonstationary and rather short time series [Gilmore, 1993, 1998].

Generally speaking, the recurrence plots are designed to locate hidden recurring patterns and structure in time series and are defined as $N \times N$ symmetric matrix:

$$R_{i,j} = \Theta(\varepsilon_i - \|\bar{x}_i - \bar{x}_j\|),\ i,j = 1, \ldots, N, \tag{1.5}$$

where $\vec{x}_{i,j}$ are phase space vectors reconstructed using Takens' time delay method. Insofar, as the RP is based on Takens' delay-coordinate embedding, when this procedure is correctly carried out, the dynamical invariants of the true and reconstructed dynamics are identical. Therefore, it is natural to assume that the RP of a correctly reconstructed trajectory bears similarity to RP of the true dynamics. In fact, $\vec{x}_i$ stands for the point in phase space at which the system is situated at time i, ε_i is a predefined cut-off distance, $\Theta(x)$ is the Heaviside function. The cut-off distance defines a sphere centered at $\vec{x}_i$. As far as recurrence of the phase space trajectory to a certain state is a fundamental property of deterministic dynamical system [Argyris, 1994; Ott, 1993; Marwan, 2002], the trajectory in the reconstructed phase space returns at time i into the ε-neighborhood of where it was at time j (i.e. if $\vec{x}_i$ is closer to $\vec{x}_j$ than the cut-off distance) $R_{i,j} = 1$ and these two vectors are considered to be recurrent. Otherwise $R_{i,j} = 0$. According to Eckman et al. [1987], the $R_{i,j}$ values can be visualized by black and white dots, but often the recurrence plot relates $R_{i,j}$ distances to a color, e.g., the longer the distance, the "cooler" the color. Thus, the recurrence plot is a solid rectangular plot consisting of pixels whose colors correspond to the magnitude of data values in a two-dimensional array and whose coordinates correspond to the locations of the data values in the array.

The black points indicate the recurrences of the investigated dynamical system revealing their hidden regular and clustering properties. By definition, RP is symmetric and has black main diagonal (the line of identity) formed by distances in matrix. In order to understand RP it should be stressed that it visualizes the distance matrix which represents autocorrelation in the series at all possible time (distance) scales. As far as distances are computed for all possible pairs, on the RP plots the elements near the diagonal correspond to short range correlation, whereas the long range correlations are revealed by the points distant from the diagonal. Hence if the analyzed dynamics (time series) is deterministic (ordered, regular), then the recurrence plot shows short line segments parallel to the main upward diagonal. At the same time, if dynamics is purely random, the RP will not present any structure at all. One of the crucial points in RP analysis is the selection of cutoff distance ε or radius. If ε is selected too low no recurrent point will be found. At the same time, it cannot be set too high as then every point will be assumed as recurrent. Exhaustive overview on this subject can be found in Zbilut [1998], Marwan [2003].

The primordial aim of RP testing was the visual inspection of structures located in high dimensional phase spaces where the above-mentioned methods are useless, especially when we deal with real data sets. The view of recurrence plots provides a unique possibility to observe time evolution patterns of phase space trajectories, both at large and short scales. According to Eckmann et al. [1987], by analysing the large scale patterns or typology, recurrence plots can be characterized as homogeneous (dynamics with uniformly distributed characteristics), periodic (dynamics with distinct periodic components), drift (dynamics with slowly varying parameters) and/or disrupted (dynamics characterized by abrupt changes). By small scale inspection, patterns (or texture) of recurrence plots can be characterized as single dots, diagonal lines, vertical lines and horizontal lines. The exact recurrent dynamics causes long diagonal lines separated by a fixed distance. A large amount of single isolated scattered dots and the vanishing amount of lines is typical for heavily fluctuating dynamics under the influence of non correlated noises (by the way, in this case insufficient dimension of embedding space is not excluded). The non regular occurrence of short as well as long diagonal lines is characteristic for low-dimensional chaotic processes, and the non regular occurrence of extended uniform areas corresponds to irregular high-dimensional dynamics. In a more general sense, the line structures in RP exhibit local time relationship between the current phase space trajectory segments. The stationarity of the whole time series requires that the density of line segments be uniform.

As far as RP was developed for single data sets, Zbilut et al. [1998] have expanded it by considering two different time series. The cross-recurrence between two series, x_i and y_i, is defined as $CR_{i,j} = \Theta(\varepsilon_i - \|\bar{x}_i - \bar{x}_j\|)$. Here, the two time series are embedded in the same phase space. The representation is analogous to RP, and it is called a cross-recurrence plot (CRP) [Marwan, 2003].

Qualitative patterns of unknown dynamics presented as fine structure of RP or CRP are often too difficult to be considered in detail. Zbilut and Webber [Zbilut, 1992] have developed a tool which quantifies the structures in RPs, namely, the Recurrence Quantitative Analysis (RQA). They define measures using the recurrence point density, the length of diagonal, and vertical structures in the recurrence plot, the recurrence rate, the entropy of recurrent points' distribution, etc. Presently at least 8 different statistical RQA values are known [Zbilut, 1992; Ivanski, 1998; Marwan, 2003], practical meaning of which is not always quite clear. Computation of these measures in small windows moving along the main diagonal of the RP reveal the time dependent behavior of these variables making it possible to identify the unknown dynamical patterns in time series [Zbilut, 1992; Marwan, 2002].

Here we will briefly touch only main RQA statistical values. The first of these statistics, termed % recurrence (%REC), is simply the percentage of points on the RP that are darkened or in other words those pairs of points whose spacing is below the predefined cut-off distance ε_i. It quantifies the number of time instants characterized by a recurrence in the signals' interaction: the more periodic the signal dynamics, the higherthe (%REC) value. Stochastic behavior causes very short diagonals, whereas deterministic behavior causes longer diagonals.

The second RQA statistic is called % determinism (%DET); it measures the percentage of recurrent points in a RP that are contained in lines parallel to the main diagonal. The main diagonal itself is excluded from these calculations because points there are trivially recurrent. Intuitively, %DET measures how ''organized'' the RP is. This variable discriminates between the isolated recurrent points and those forming diagonals. Since a diagonal represents points close to each other, successively forward in time, DET also contains the information about the duration of a stable interaction: the longer the interactions, the higher the DET value. Stochastic and heavily fluctuating data cause none or only short diagonals, whereas deterministic systems cause longer diagonals.

The third often used RQA statistics, called entropy (ENT), is closely related to %DET. ENT is Shannon information entropy of line distribution measured in bits and is calculated by binning the diagonal lines according to their lengths and using the following formula:

$$\mathrm{ENT} = -\sum_{k=1}^{N} P_k \log_2 P_k$$

where N is the number of bins and P_k is the percentage of all lines that fall into bin k. In other words, P_k is defined as the ratio between the number of k-point long diagonals, and the total number of diagonals. ENT is measured in bits of information, because of the base-2 logarithm. Thus, whereas DET accounts for the number of the diagonals, ENT quantifies the distribution of the diagonal line lengths.

The more different the lengths of the diagonals, the more complex the deterministic structure of the RP. A more complex dynamics will require a larger number of bits (ENT) to be represented.

The fourth RQA statistics, termed TREND, measures how quickly a RP goes away from the main diagonal. As the name suggests, TREND is intended to detect nonstationarity in the data. The fifth RQA statistics is called length of the maximal deterministic line (MAXLINE) and is equal to the reciprocal of the longest line length found in the computation of DET, or $1/line_{\max}$. Eckmann, Kamphorst, and Ruelle claim that line lengths on RPs are directly related to the inverse of the largest positive Lyapunov exponent [Eckmann et al., 1987]. Relatively small $line_{\max}$ values are therefore indicative of chaotic behavior. In a purely periodic signal, there is an opposite extreme, lines tend to be very long, so MAXLINE is very small.

The RQA technique gives a local view of the studied time series, based on the single distance pairs in phase space and is suited for the detection of changes of analyzed dynamics. This method is the most comfortable for qualitative discrimination between signals and random noise.

1.3 Investigation of dynamics of complex natural process: Caucasus seismicity

The significant variability exhibited both in time and in space makes the problem of identification and quantification of geophysical phenomena extremely complicated. Therefore, the best way to understand dynamical features of complex geophysical processes is to analyse the measured data sets using modern nonlinear methods.

Earthquakes are expression of the continuing evolution of the planet Earth and of the deformation of its crust. Dynamics of seismic processes is viewed as extremely complicated, so that the level of "turbulence" of the lithosphere exceeds that of the atmosphere [Kagan, 1992, 1994, 1997].

During more than one hundred years of instrumental observations, several important characteristics of spatial, temporal and energetic distributions of earthquakes have been revealed [Scholz, 1990; Keilis-Borok, 1990; Turcotte, 1992; Goltz, 1997; Matcharashvili, 2000; Rundle, 2000]. Nevertheless, the question of dynamics of seismic processes remains the subject of intense discussions because it is directly tied with the problem of earthquake prediction. Opponents of earthquake prediction [Kagan, 1992, 1994, 1997; Kanamori, 2001; Geller, 1999; Ben-Zion, 2008 etc.] regard seismic processes as completely random while proponents assume them as complex and high-dimensional though not random [Main, 1997; Wyss, 1997; Chelidze, 1997; Knopoff, 1999]. Indeed, completely random processes are unpredictable on any spatial and temporal scales. On the other hand, in processes with nonrandom dynamical structure there always exist specific spatial and temporal scales for which the system is close to deterministic, i.e., it is predictable at least for a not very far future. From this point of view, if seismic process has a nonrandom structure it could not be regarded as unpredictable. Of course, it is clear that predictability in this sense does not necessarily mean "real" forecast of every hazardous event in practically meaningful time scales. At present, evidence of nonrandom structure of seismicity has mainly scientific importance because it gives ground to efforts aimed at finding predictive markers. This is also important for modern ideas on possible control of practically unpredictable seismic processes. To bring some light into this problem, we consider dynamic structure of seismic process in Caucasus.

As mentioned above, one of the most popular approaches to the problem of identification of patterns of complex dynamics, including seismicity, is based on the evaluation of nonlinear structures (or, just the same, of nonlinear structures of appropriate time series) [Theiler et al., 1992; Rapp et al., 1993]. In this way, it is possible to achieve reliable detection of dynamical regime(s) of seismic process by calculating their measurable characteristics. These characteristics can be calculated for a general seismic process as well as prior to and after strong earthquakes. This is important in search of possible earthquake predictive dynamical markers. It is known that from both, qualitative and quantitative points of view, seismological data bases are as a rule not sufficient for proper nonlinear evaluation of lithospheric dynamics, even for relatively low-dimensional processes. Therefore, similar to other fields, evaluation of nonlinear structure of geophysical data seems to be a more appropriate approach.

In order to answer the above question on the dynamical characteristics and nonlinear structure of earthquake generation it is necessary to investigate dynamical properties of seismic processes in all three domains: energetic, spatial and temporal. For this purpose, "time series" of inter-event time intervals (waiting times), magnitude sequences and inter-event distances, have been analyzed for earthquakes in Caucasian region. Analyzed were also similar time series of smaller regions of Greater Caucasus and Javakheti in1962–1993. All these time series were taken from the earthquake catalogue for the Caucasus and the adjacent territories of Northern Turkey and Northern Iran for the 1962–1993 time period (Seismological Data Base of Institute of Geophysics, Tbilisi, Georgia).

It was shown that despite the fact that the size and temporal distributions of earthquakes obey a power law, they are dynamically quite different. The magnitude distribution of earthquakes in the Caucasian region is undoubtedly high-dimensional, d_2 as a rule is larger than 8 ($d_2 > 5$ is assumed as a high dimensionality threshold) [Sprott, 1997]. According to our results as well as reports of other authors [Sadovsky, Pisarenko, 1991; Korvin, 1992] the fractal dimension for the distribution of inter-earthquake distances is low ($d_2 < 2$). Most interesting is that the waiting times distribution reveals an obviously low dimensional nonlinear structure (d_2 of the order of 1.6–2.5 and λ_{max} of the order of 0.2–0.7), although it can not be recognized as a deterministic chaos [Matcharashvili, 2000] (see Fig. 1.1). The low dimensionality of earthquakes temporal distribution is in complete agreement with earlier results for other parts of the globe [Goltz, 1998].

The next main goal of investigation was a qualitative evaluation of earthquakes′ time and size distribution peculiarities, taking place before and after strongest regional events as well as quantitative discrimination of dynamical characteristics preceding and following largest regional earthquakes.

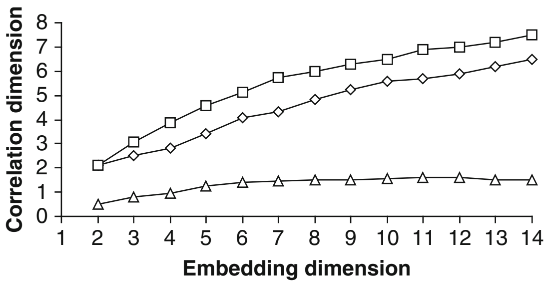

Fig. 1.1 Typical plot of correlation dimension d_2 versus embedding dimension p for the Caucasus, the Greater Caucasus, and the Javakheti region, magnitude (middle curve) and waiting times (lower curve) sequences. The upper curve represents the random numbers set

So as a next step on the way to a better understanding of the underlying dynamics of earthquake generation, we have undertaken comparison of the properties of waiting time distribution before and after large events. For this purpose we have considered waiting time sequences of a seismic catalogue, separately before and after the largest events, using the above-mentioned tests such as correlation dimension, Lyapunov exponent calculation as a measure of non-linearity.

We investigated dynamical characteristics of seismic processes before and after four earthquakes of the Caucasian region (Daghestan, Paravani, Spitak and Racha) that were the strongest in the considered period.

According to the results of our analysis, the general properties of dynamics of earthquakes temporal distribution before and after the largest regional events do not indicate a qualitative difference from the integral dynamics obtained by consideration of time series from the whole original catalogues [Matcharashvili, 2002, 2007]. Indeed, correlation dimensions of all the considered waiting time sequences from the original catalogue (containing all independent events and aftershocks above the threshold magnitude), both preceding and following the largest events in the Caucasus, converge to a limit value. At the same time it is important that these values are not coinciding. Consequently, as long as all the investigated time series have correlation dimension lower than the low dimensional threshold ($d_2 < 5$) [see also Goltz, 1998], it can be deduced that the temporal distribution of earthquakes is characterized by a low-dimensional dynamics before, as well as after the largest regional events. At the same time, in the energetic domain earthquakes' magnitude distribution remains high-dimensional before and after strong events. As it was stressed above, the results of dimensional calculations, especially when a low-dimensional process is detected, should be verified using special methods.

While testing low-dimensional interevent time sequences, we have typical problems, always encountered in testing real, usually short and noisy time series. As it was already mentioned, in order to overcome discriminating problems, as in the case of high-dimensional processes, one has to test time series for the evidence of nonlinearity [Theiler and Prichard, 1997]. One additional reason why this approach becomes popular, is that from the practical point of view the goal of detecting nonlinearity in low dimensional data is easier than a confident identification of chaotic dynamics [Theiler, 1992].

It was found that in all cases the time interval sequences obtained from the original catalogue above the threshold magnitudes before and after the largest events reveal evidence of a nonlinear structure. In other words, the null hypothesis that these sequences are generated by linearly correlated noise or by static monotone nonlinearity should be rejected. The significance of differences of **S**-measure of natural sequences before and after the earthquakes considered from the appropriate phase randomized ($\mathbf{S}_{PR}$) and Gaussian scaled random phases ($\mathbf{S}_{GSRP}$) surrogates are significant at $p < 0.005$ confidence level; thus the significance of differences for waiting time sequences before and after Dagestan (M = 6.6) earthquake are $\mathbf{S}_{PR} = 55.6 \pm 0.27$, $\mathbf{S}_{GSRP} = 15.9 \pm 0.20$ and $\mathbf{S}_{PR} = 50.5 \pm 0.15$, $\mathbf{S}_{GSRP} = 17.1 \pm 0.13$; for Paravani (M = 5.6) earthquake $\mathbf{S}_{RP} = 51.1 \pm 0.21$, $\mathbf{S}_{GSRP} = 16.2 \pm 0.13$ and $\mathbf{S}_{PR} = 64.2 \pm 0.27$, $\mathbf{S}_{GSRP} = 11.5 \pm 0.17$; for Spitak (M = 6.9) earthquake $\mathbf{S}_{PR} = 49.2 \pm 0.12$, $\mathbf{S}_{GSRP} = 11.4 \pm 0.12$ and $\mathbf{S}_{PR} = 52.2 \pm 0.27$, $\mathbf{S}_{GSRP} = 15.2 \pm 0.19$; for Racha (M = 6.9) earthquake $\mathbf{S}_{PR} = 57.6 \pm 0.23$, $\mathbf{S}_{GSRP} = 16.3 \pm 0.23$ and $\mathbf{S}_{PR} = 51.5 \pm 0.17$, $\mathbf{S}_{GSRP} = 18.4 \pm 0.11$.

Besides, for a nonlinear structure testing, the RQA method is suitable for short seismic data sets. As shown in Figs. 1.2 and 1.3, the extent of the order in magnitude distribution of Caucasian earthquakes before and after M6.9 Racha earthquake has been noticeably changed. Strictly speaking, the energetic distribution becomes more regular while the temporal distribution becomes essentially irregular. It is

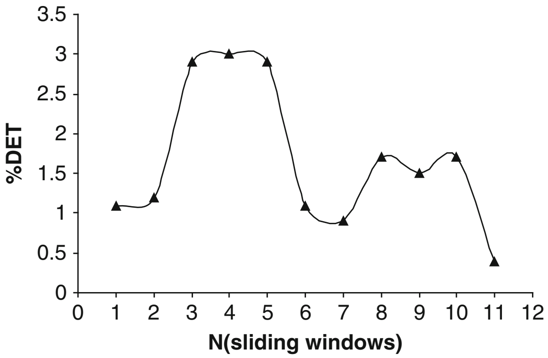

Fig. 1.2 RQA measure, %DET, for Racha (M6.9) earthquake magnitude sequences. Consecutive non overlapping 500 data width sliding windows (M6.9 event occurred in 5-th window, and M6.2 in 8-th window)

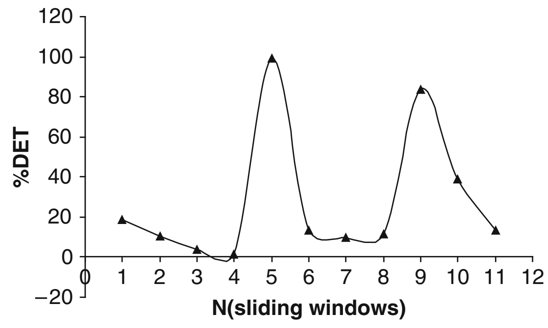

Fig. 1.3 RQA measure, %DET, for Racha (M6.9) earthquake inter-event time intervals sequences. Consecutive non-overlapping 500 data width sliding windows (M6.9 event occurred in 5-th window, and M6.2 in 8-th window)

worth to mention that a decrease of the order in earthquake temporal distribution is distinctive for both strong Racha earthquakes, M6.9 and its aftershock M6.2. At the same time, an increase of the order in energetic distribution is not so clear.

To understand the above-mentioned differences in the correlation dimension values before and after largest earthquakes, we used a sliding windows technique. We considered a sequence of 6695 events of Paravani earthquakes inter-event time intervals. Here No = 5300 is the ordinal number of the time interval which directly precedes the largest earthquake. We have calculated d_2 for 1000 event sliding windows with a step of 50 events starting with event No = 3200 up to event No = 5800. Hence, the first window consists of time interval sequences between earthquakes in the range of ordinal numbers 3200–4200. As shown in Fig. 1.4, values of d_2 decrease for the windows following the largest event. The decrease begins when a sliding window contains about 20 inter-event time intervals after the largest event, and becomes significant when 40–50 such events are included in the sequence. Note that the window 4310–5310, like the window 4300–5300, reveals the background value of a correlation dimension for waiting time sequences before the largest earthquake. It seems doubtful to expect that such an essential change in the dynamical properties of the considered sequence could have been caused by the addition of so few new data, unless there is a hidden regularity in the sufficiently long waiting time sequence containing data preceding the largest event.

Next we used RQA approach to further quantify dynamical changes in earthquakes energetic and temporal distributions in Javakheti region. As shown in Figs. 1.5 and 1.6, dynamical changes in earthquakes energetic and spatial distributions detected before and after Paravani earthquake are very similar to those found before and after the Racha strong earthquake.

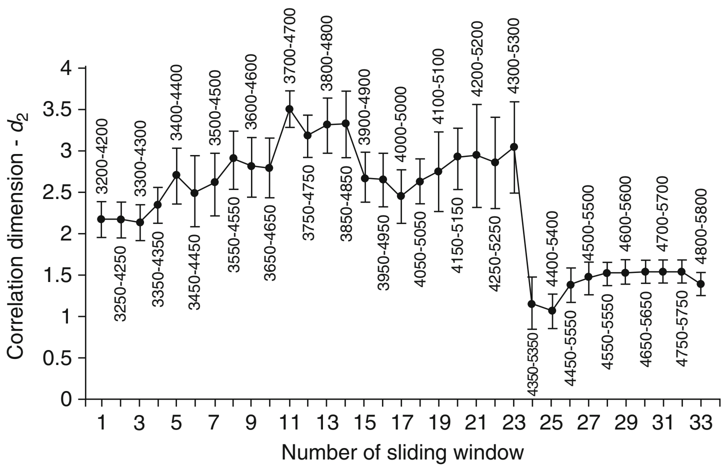

Fig. 1.4 Variation of correlation dimension d_2 of waiting time sequences calculated for sliding windows containing 1000 consecutive time intervals at 50 events step (Paravani earthquake epicentral area)

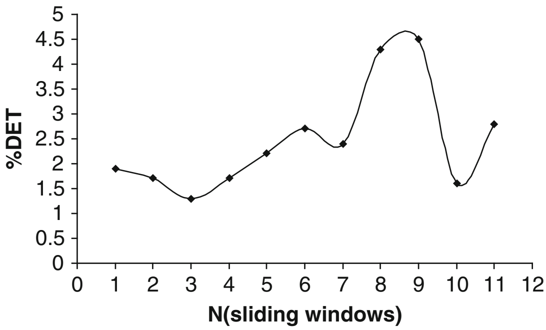

Fig. 1.5 RQA measure, %DET, for Paravani (M5.6) earthquake magnitude sequences. Consecutive non overlapping 600 data width sliding windows (M5.3 event occurred in 6-th window, M5.6 in 9-th window, and M5.1 in 11-th window)

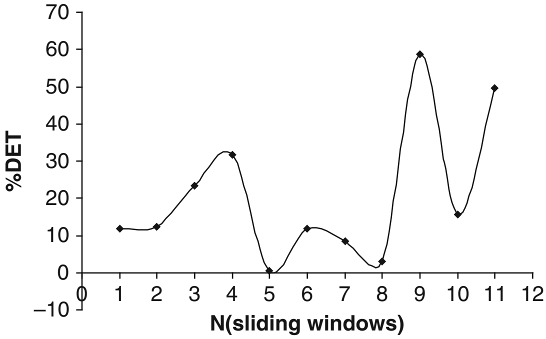

Fig. 1.6 RQA measure, %DET, for Paravani (M5.6) interevent time intervals sequences. Consecutive non overlapping 600 data width sliding windows (M5.3 event occurred in 6-th window, M5.6 in 9-th window, and M5.1 in 11-th window)

Indeed, magnitude distribution before the M5.6 earthquake becomes more regular while temporal distribution becomes noticeably irregular. Also, similar to the Racha earthquake, changes were observed before the M5.1 earthquake which may be regarded as an aftershock; namely a decrease in the order of temporal distribution, while an increase of the order in magnitude distribution is not clear. It is interesting to mention that the same situation was observed for M5.3 earthquake preceding the main M5.6 event.

These results indicate that measuring of dynamical characteristics of seismic time series may provide markers having in future a precursory value which may help in developing modern earthquake prediction approaches [Matcharashvili et al. 2002].

Thus it is clear that seismicity in two domains (temporal and spatial) out of three (energetic, temporal and spatial) reveals low-dimensional nonlinear structure. This and similar results lead to understanding that in spite of extreme complexity, the processes related to the earthquake generation are characterized by some internal dynamic structure and thus are not completely random [Smirnov, 1995; Goltz, 1998; Rundle et al. 2000; Matcharashvili et al. 2002]. Despite the proofs that seismic activity is a non-random process, the physics of internal or external factors involved is still poorly understood, but it can be asserted that the general problem of earthquake prediction and/or earthquake triggering, one of the most challenging targets of nowadays science, should not be further considered as an "alchemy of present time" [Geller, 1999]. In other words, the quest for earthquake predictive markers or triggering factors should be recognized as obviously difficult, though scientifically well grounded task related to the search for determinism in the complex seismic process.

References

Abarbanel, H. D. I., Brown, R., Sidorowich Tsimring, L. S., Rev. Mod. Phys., 65, 4, 1331–1392, 1993.

Antani, J.A., Wayne, H.H., Kuzman, W.J., Am. J. Cardiol. 43, 2, 239–247, 1979.

Arecchi, F.T., Fariny, Lexicon of Complexity, A., ABS. Sest. F. Firenze, 1996.

Argyris, J. H., Faust, G., Haase, M., An Exploration of Chaos, North-Holland, Amsterdam, 1994.

Bak, P., Tang, C., Wiesenfeld, K. Phys.Rev. A., 38, 364–374. 1988.

Beeler, N.M., Lockner, D.A., J. Geophys. Res., 108: ESE 8,1–17. 2003.

Bennett, C. H., in *Complexity, Entropy and the Physics of Information*, edited by W. H. Zurek, Addison-Wesley, Reading, MA, 1990.

Ben-Zion, Y. Collective Behavior of Earthquakes and Faults. Rev. of Geophysics, 46, doi: 10.1029/2008RG000260

Berge, P., Pomeau, Y., Vidal, C., Order within chaos. J. Wiley, NY, 1984

Bhattacharya, J., In Search of Regularity in Irregular and Complex Signals, PhD Thesis, 1999.

Boffetta, G., Cencini, M., Falcioni, M., Vulpiani, A., Phys. Rep. 356, 367, 2001.

Bowman D., Ouillon G., Sammis C., Sornette A., Sornette, D., J. Geophys. Res. 103: 24359–24372. 1998.

Casdagli, M.C. Physica D 108, 12–44, 1997.

Castro, R., Sauer, T., Phys. Rev. E 55, No 1, 287–290, 1997.

Chelidze, T., Matcharashvili, T., Computers and Geosciences, 2003, 29. 5. 587–593.

Chelidze, T., Matcharashvili, T., Gogiashvili, J., Lursmanashvili, O., Devidze, M., Nonlinear Processes in Geophysics, 2005, 12, 1–8.

Cover T. M., J. A. Thomas, *Elements of Information Theory* (Wiley, New York, 1991).

Eckman, J.P., Kamphorst, S.O., Ruelle, D., Recurrence Plots of Dynamical Systems, *Europhys. Lett.*, **4**, 973–977, 1987.

Elbert, T., Ray, W.J., Kowalik, Z.J., Skinner, J.E., Graf, E. K., Birnbauer, N., Physiol Rev. v 74. 1-49, 1994.

Garfinkel, A., Chen, P., Walter, D.O., Karaguezian, H., Kogan, B., Evans, S.J., Karpoukhin, M., Hwang, C., Uchida, T., Gotoh, M., Weiss, J.N., J. Clin. Invest. 99, 2, 305–314, 1997.

Gavrilenko, P., G, Melikadze., Chelidze, T., Gibert, D., Kumsiashvili, G., Geophys. J. Int., 2000,143, 83–98.

Geller, R.J., Earthquake prediction: is this debate necessary? Nature, Macmillan Publishers Ltd 1999 Registered No. 785998 (http://helix.nature.com/debates).

Gilmore, R., J. Econ. Behav. Organization, 22, 1993, 209–237.

Gilmore, R., Rev. Mod. Phys. 70, 1455–1529, 1998.

Goltz C, Fractal and chaotic properties of earthquakes, Springer, Berlin, 1998.

Govindan, R.B., Narayanan, K., Gopinathan, M.S., Chaos, 8, 2, 495–502, 1998.

Grassberger, P., Procaccia, I., Rev. A. 28, 4, 2591–2593, 1983.

Hegger, R., Kantz, H., Schreiber, T. Chaos **9**, 413–440, 1999.

Hodgson, Geoffrey M.. Economics and Evolution: Bringing Life Back into Economics. Ann Arbor: University of Michigan Press. 1993

Ivanski, J., E. Bradley, Chaos **8**, 861 1998
Johansen, A., Sornette, D., Phys. Rev. Lett. 82: 5152–5155. 1999.
Jones, N., New Scientist. June 30, 34–37. 2001.
Kagan, Y.Y., Physica D77, 160–192, 1994.
Kagan, Y.Y., Geophys. J. Int., 131, 505–525, 1997.
Kanamori, H., Brodsky, E.E., Physics Today, 2001, 6, 34–40.
Kantz, H., Schreiber, T., Nonlinear time series analysis, Cambridge, University Press, 1997.
Keilis-Borok, V.I., Physica D, 77, 193–199, 1994.
Kennel, MB, Brown R and Abarbanel, HDI, Phys. Rev. A 45, 3403–3411, 1992
King, C. Y., Azuma, S., Igarashi, G., Ohno, M., Saito, H., Wakita, H., J. Geoph. Res. 104, B6, 13073–13082, 1999
Knopof, L., Earthquake prediction is difficult but not impossible, Nature, Macmillan Publishers Ltd 1999 Registered No. 785998
Korvin, G., Fractal models in the earth sciences, Elsevier, NY, 1992.
Kraskov, A., Stögbauer, H., Grassberger, P., Phys. Rev. E, 69, 066138, 2004.
Kumpel, H., Evidence for self-similarity in the harmonic development of earth tides, in: Kruhl, J.H. (Ed), Fractals and dynamic systems in geoscience, Springer, Berlin,, 213–220. 1994.
Lefebvre, J.H., Goodings, D.A., Kamath., Fallen, E.L., Chaos, 3. 2. 267–276. 1993.
Main, I., Nature, 1997, 385, 19–20.
McCauley, Joseph L. 2004. *Dynamics of Markets:* Econophysics and Finance. Cambridge, UK: Cambridge University Press.
Mason, D. T., Spenn, J.F., Zelis, R., Am. J. Cardiol. 26. 3. 248–257. 1970.
Marzochi, W., Physica D, 90, 31–39, 1996.
Marwan, M., Wessel, N., Meyerfeldt, U., Schirdewan, A., Kurths, J., Phys. Rev. E., 66, 026702, 2002.
Marwan, M., Encounters with neighborhood, PhD Thesis, 2003.
Matcharashvili, T., Chelidze, T., Javakhishvili, Z., Nonlinear Processes in Geophysics, 7, 9–19. 2000.
Matcharashvili, T., Janiashvili, M., Sulis, W., Trofimova, I., (Eds), Nonlinear dynamics in life and social sciences. IOS Press, Amsterdam, 2001, 204–214.
Matcharashvili, T., Chelidze, T., Javakhishvili Z., Ghlonti, E. Computers & Geosciences 2002, 28. 5. 693–700.
Ott, E., Chaos in Dynamical Systems, Cambridge University,Press, Cambridge, 1993.
Packard, N.H., Crutchfield, J.P., Farmer, J.D., Shaw, R.S., Phys. Rev. Lett. 45, 712–716, 1980.
Peinke, J., Matcharashvili, T., Chelidze, T., Gogiashvili, J., Nawroth, A., Lursmanashvili, O., Javakhishvili, Z.,Influence of Periodic Variations in Water Level on Regional Seismic Activity Around a Large Reservoir: Field and Laboratory Model Phys. Earth. Planet. Int. 2006, (1–2), 130–142
Pikkujamsa, S., Makikallio, T., Sourander, L., Raiha, I., Puukka, P., Skytta, J., Peng, C. K., Goldberger, A., Huikuri, H., Circulation, 100, 4, 393–399, 1999.
Rapp, P.E., Albano, A.M., Schmah, T.I, Farwell, L. A., Phys. Rev. E., 47, 4, 2289–2297, 1993.
Rapp, P.E., Albano, A.M., Zimmerman, I. D., Jumenez-Montero, M.A., Phys. Lett. A., 192, 1, 27-33, 1994.
Rapp, P. E., Cellucci, C. J., Korslund, K. E., Watanabe, T. A., Jimenez-Montano, M. A., Phys. Rev. E **64**, 016209, 2001.
Ruelle, D., Physics Today, 47, 7, 24–32, 1994
Rombouts S.A., Keunen, R.W., Stam, C.J., 1995. Phys.Lett. A. 202.5,6.352–358.
Rosenstein, M. T., Collins, J. J., DeLuca, C. J., Physica D, **65**, 117–134,1993.
Rundle, J., Turcotte, D., Klein, W., (Editors), 2000. GeoComplexity and the physics of earthquakes, American Geophysical Union, Washington.
Sato, S., Sano, M., Sawada, Y., Prog. Theor. Phys. 77, 1–5, 1987.
Schreiber, T., Phys. Rep. 308,1–64, 1999
Schreiber, T., Schmitz, A., Phys. Rev. E 59, 044, 1999.

Schreiber, T., Phys. Rev. E, 1993, 47, 4, 2401–2404.
Shannon, C. E., Bell Syst. Tech. J. **27**, 379 (1948); **27**, 623, 1948.
Shannon, C. E., *The Mathematical Theory of Communication*, University of Illinois, Urbana, IL, 1964.
Shiner, J. S., Davison, M., Landsberg, P. T., Phys. Rev. E, 59, 2, 1459–1464, 1999.
Scholz C.H., Nature, 1990, 348, 197–198.
Sibson, R., 1994. Crustal stress, faulting and fluid flow, in Parnell J. (Ed), Geofluids: Origin, Migration and Evolution of Fluids in sedimentary Basins. The Geological Society, London, 69-84.
Simpson, D.W., Leith, W. S., Scholz, C., 1988. Bull. Seism. Soc. Am. 78: 2025–2040.
Smirnov, V.B., J. of Earthq. Prediction Res., 1995, 4, 31–45.
Sivakumar B, Berndtsson R, Olsson J, Jinno K. Hydrol. Sci. J. 47, 1, 149–58, 2002
Sprott, J. C., Rowlands, G., Chaos data analyzer; the professional version. AIP, NY, 1995.
Takens, F., ''Detecting strange attractors in fluid turbulence,'' in *Dynamical Systems and Turbulence*, edited by D. Rand and L.-S. Young, Springer, Berlin, 1981, pp. 366–381
Talwani, P., Pure and appl. geophys, 150: 473–492. 1997.
Tarasov, N.T. Transactions (Doklady) of the Russian Academy of Sciences, 353A (3), 445–448, 1997.
Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., Farmer, J.D., Physica, D58,77–94, 1992.
Theiler, J., Prichard, D., Using "Surrogate-surrogate data" to calibrate the actual rate of false positives in tests for nonlinearity in time seies. in "Nonlinear dynamics and Time Series" eds. Cutler, D., Kaplan, D.T., 11, 99–113, Fields Institute Communications, 1997.
Turcotte, D., Fractals and chaos in geology and geophysics. University Press, Cambridge, 1992.
Vidale, J. E., D. C. Agnew, M. J. S. Johnston, and D. H. Oppenheimer, J. Geophys. Res., 103, 24,567–24,572. 1998.
Volykhin, A.M., Bragin, V.D., Zubovich, A.P., Geodynamic Processes in Geophysical Fields, Moscow: Nauka, 1993.
Wackerbauer, R., Witt, A., Atmanspacher, H., Kurths, J., Scheingraber, H., Chaos, Solitons Fractals **4**, 133,1994.
Weiss, J.N., Garfinkel, A., Karaguezian, H., Zhilin, Q., Chen, P., Circulation, 99(21), 2819–2826, 1999.
Wolf, A., Swift, J., Swinney, H., Vastano, J., Physica D, 16, 285–317, 1985.
Wyss, M., Science, 278, 487–488. 1997
Yang, P., Brasseur, G. P., Gille, J. C., Madronich, S., Physica D76, 3310343, 1994.
Yao, W., C, Essex., Yu, P., Davison, M., Phys. Rev. E. 69, 110–123, 2004
Zbilut, J.P., Weber, C, L., Phys. Lett. A 171, 199–203, 1992
Zbilut, J.P., A. Giuliani, C.L. Webber Jr., Phys. Lett. A 246 (1998) 122–128.
Zhang, X., Thakor, N.V., IEEE Trans. on Biomed. Eng. 46, 5. 548–555. 1999
Zokhowski, M., Winkowska-Nowak, K., Nowak, A., Phys. Rev. E. 56, 3, 3725–3727, 1997.