Chapter 3
Shear Oscillations, Rotations and Interactions in Asymmetric Continuum

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Abstract A concise asymmetric continuum theory including the relations between stresses, strains, interaction fields and defects is presented. In the presented theory, the motion equations for antisymmetric part of stresses replace the balance of angular momentum. Considering the symmetric stresses, we present a new form of the motion equations for the deviatoric part of strains, arriving at the definition of shear-twist motion as the oscillation of the axes of shears and their amplitudes. With the help of Dirac tensors we present an invariant form of these motions. The motions – displacement and rotations – generated in source processes, e.g., in an earthquake source, may be generated independently or with some phase shift due to the rebound processes; therefore, in the presented asymmetric continuum theory we introduce the phase shift index between the strains and rotations. The presented invariant system of motion equations makes it possible to obtain solutions with the simultaneous strains and rotation motion or those with the $\pi/2$ phase shift between them.

Further, we include in this asymmetric theory, besides the mechanical system, some interaction fields, e.g., thermal and electric interaction.terms. The presented interaction theory is equivalent to that given by Kröner, but it is practically much simpler and includes new solutions with the simultaneous strains and rotation motions or those with the phase shift between them.

3.1 Introduction

We present some elements of the asymmetric continuum theory with some important applications; our consideration on the asymmetric continuum theory includes:

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– balance laws for the symmetric and antisymmetric stresses and related wave fields
– fundamental relations between the asymmetric stresses and dislocation fields
– hypothesis of a synchronization process based on the rebound processes and the wave
– solution with the phase shift between strains and rotations
– interaction of physical fields with a mechanical system; our consideration is limited to the thermal and electric interaction.terms.

The presented theoretical study generalizes those presented by Teisseyre (2008), Teisseyre (2009), and Teisseyre and Gorski (2009).

3.2 Asymmetric Continuum

Our asymmetric theory differs essentially from the other approaches; e.g., the theory of asymmetric elasticity founded by Nowacki (1986); it includes the couple-stresses introduced in a similar way as in the micropolar and micromorphic theories (see: Eringen, 1999).

A search to improve the classic continuum theory is based on the numerous defaults of the classic theory. We can add here one more example of such defaults, as pointed out by Roux and Guyon (1985). Those authors compared various numerical simulations with the experimental data related to electric and mechanical coupling; some especially poor results concern the cases in which the momentum effects play an essential role. The authors suggest that the angular elasticity should be taken into account. Making the reference to the publication by Crandall et al (1978), they suggest that the elastic energies related to normal and shear forces should be supplemented by the terms including the flexion torque and torsion torque when constructing a more general definition of the Hamiltonian.

Our version of the asymmetric theory includes the asymmetric stresses, symmetric strains and rotations; it permits to include the phase shift between the displacement and rotation motions. As regards the constitutive laws joining the antisymmetric stresses and rotations we follow some ideas introduced by Shimbo (1975; 1995) and related consideration on the friction processes and rotation of grains.

We have constructed our theory (Teisseyre, 2009) as based on the asymmetric stresses, S_{kl}, and deformations: symmetric strains, E_{kl}, and antisymmetric rotations, ω_{kl}:

$$S_{kl} = S_{(kl)} + S_{[kl]}, \quad E_{kl} = E_{(kl)}, \quad \omega_{kl} = \omega_{[kl]} \tag{3.1}$$

We underline that the deformation energy becomes related also to rotation motions:

$$E = \frac{1}{2} S_{kl}(E_{kl} + \omega_{kl}) = \frac{1}{2} S_{(kl)} E_{kl} + \frac{1}{2} S_{[kl]} \omega_{kl}$$

Instead of the Kröner method (Kröner, 1981) based on the self-fields, we introduce the material structure indexes, $e^0$and χ^0, which may help us to join the deformation fields, strains and rotations, with the observed displacement motions:

$$\begin{aligned} E_{kl} &= e^0 E_{kl}^0 = e^0 \frac{1}{2}\left(\frac{\partial}{\partial x_k} u_l + \frac{\partial}{\partial x_l} u_k\right), \\ \omega_{kl} &= \chi^0 \omega_{kl}^0 = \chi^0 \left[\frac{\partial}{\partial x_k} u_l - \frac{\partial}{\partial x_l} u_k\right] \end{aligned} \tag{3.2}$$

For $e^0 = 1$and $\chi^0 = 0$, we return to classic elasticity, while for $e^0 = 0$ and $\chi^0 = 1$ we will have a continuum with rigid, densely packed spheres with friction sensitive to an external moment load. The independent fields $(E_{kl},\ \omega_{kl})$ lead us to defects and extreme deformations.

In our theory, for solid elastic bodies we put:

$$e^0 = 1, \qquad E_{kl} = E_{kl}^0, \qquad \omega_{kl} = \chi^0 \omega_{kl}^0 \tag{3.3}$$

where the phase index χ^0 may vary from 0 to ± 1 or $\pm$i.

The Shimbo (1975) consideration helps us to present the constitutive relations:

$$S_{(kl)} = \lambda \delta_{kl} E_{ss} + 2\mu E_{lk}, \quad S_{[kl]} = 2\mu \omega_{kl}, \quad S_{(kl)}^D = 2\mu E_{kl}^D, \tag{3.4}$$

where symbols $S_{(kl)}^D$ and E_{kl}^D mean the respective deviatoric tensors, e.g., $S_{(kl)}^D = S_{(kl)} - \frac{1}{3}\delta_{kl} S_{ss}$.

Now, we can consider the motion equations for asymmetric stresses (Teisseyre, 2009). The motion equation for the symmetric part of stresses, $\partial S_{(kl)}/\partial x_k = \rho \partial^2 u_l / \partial t^2 + F_l - \partial p/\partial x_l$, leads to the relation:

$$\frac{\partial^2}{\partial x_n \partial x_l} \lambda E_{ss} + \mu \left(\frac{\partial^2}{\partial x_k \partial x_k} E_{nl} + \frac{\partial^2}{\partial x_l \partial x_n} E_{ss}\right) = \rho \frac{\partial^2}{\partial t^2} E_{nl} + \frac{1}{2}\left(\frac{\partial F_n}{\partial x_l} + \frac{\partial F_l}{\partial x_n}\right) - \frac{\partial^2}{\partial x_n \partial x_l} p \tag{3.5}$$

This expression can be divided into the wave equations for the axial and deviatoric strains:

$$(\lambda + 2\mu) \frac{\partial^2}{\partial x_k \partial x_k} E_{ss} - \rho \frac{\partial^2}{\partial t^2} E_{ss} = -\frac{\partial^2}{\partial x_k \partial x_k} p \quad \text{at} \quad \frac{\partial}{\partial x_s} F_s = 0 \tag{3.6}$$

$$\begin{aligned} \mu \frac{\partial^2 E_{nl}^D}{\partial x_k \partial x_k} - \rho \frac{\partial^2 E_{nl}^D}{\partial t^2} = -(\lambda + \mu)\left(\frac{\partial^2 E_{ss}}{\partial x_n \partial x_l} - \frac{\delta_{nl} \partial^2 E_{ss}}{3 \partial x_k \partial x_k}\right) + \frac{1}{2}\left(\frac{\partial F_n}{\partial x_l} + \frac{\partial F_l}{\partial x_n}\right) \\ - \left(\frac{\partial^2 p}{\partial x_n \partial x_l} - \frac{\delta_{nl} \partial^2 p}{3 \partial x_k \partial x_k}\right) \end{aligned} \tag{3.7}$$

We shall note that in Teisseyre (2008 and 2009) the last relation was presented with some mistakes.

The balance relation for the antisymmetric stresses $S_{[ni]}$ can be deduced from the balance of the stress moment (Teisseyre, 2009):

$$\frac{1}{l^2}\frac{\partial M_{lk}}{\partial x_k} = \varepsilon_{lki}\frac{\partial^2}{\partial x_k \partial x_n}S_{[ni]} = \rho\varepsilon_{lki}\ddot{\omega}_{ki} + \varepsilon_{lki}K_{[ki]} = \rho\varepsilon_{lki}\ddot{\omega}_{ki} + \frac{1}{2}\left(\frac{\partial F_i}{\partial x_k} - \frac{\partial F_k}{\partial x_i}\right) \quad (3.8)$$

where l is the characteristic Cosserat length, $K_{[ki]}$ is a couple of external forces and an angular moment, M_{lk}, is defined as the gradient of the antisymmetric stresses, $M_{lk} = \varepsilon_{lki}\frac{\partial}{\partial x_n}S_{[ni]}$.

For the balance law we can write now:

$$\frac{\partial}{l^2 \partial x_k}M_{lk} = \frac{\varepsilon_{lki}\partial^2}{\partial x_k \partial x_n}S_{[ni]} = \frac{\varepsilon_{lki}\partial^2}{\partial x_n \partial x_n}S_{[ki]} = \rho\varepsilon_{lki}\ddot{\omega}_{ki} + \varepsilon_{lki}K_{[ki]},$$

or

$$\mu\,\Delta\omega_{ki} - \rho\ddot{\omega}_{ki} = K_{[ki]} \quad (3.9)$$

where the transformation we made, $\frac{\varepsilon_{lki}\partial^2\omega_{ni}}{\partial x_k \partial x_n} \to \frac{\varepsilon_{lki}\partial^2\omega_{ki}}{\partial x_n \partial x_n}$, is valid for any antisymmetric non-source fields, $\partial\omega_s/\partial x_s = 0$ (where $\omega_l = {}^1\!/_2\varepsilon_{lki}\omega_{ki}$) and at the compatibility condition $\varepsilon_{imk}\varepsilon_{jns}\frac{\partial^2}{\partial x_m \partial x_n}\omega_{ks} = 0$.

The final relation (3.9) replaces that for the stress moment.

Experimental evidences for the appearance of rotation and shear oscillation (sometimes called the shear-twist) in a seismic field is based on the records of seismic rotation fields (see: Teisseyre at al.(eds), 2006; Teisseyre, 2009, Teisseyre K.P., 2007).

3.3 Rotation and shear-twist motions

The rotation motion is governed by equations (3.9), while relation (3.7) for the shear deviatoric strain, E^D_{ik}, transformed to its off-diagonal form, achieved in a special coordinate system, may be replaced by the shear-twist pseudo-vector, $\tilde{E}_s$:

$$\{\tilde{E}_s\} = \{E^D_{23},\ E^D_{31},\ E^D_{12}\} \quad (3.10)$$

However, we can maintain this definition as an invariant form for any system with the help of the Dirac tensors; the 4D invariant tensor, $\tilde{E}_{\lambda\kappa}$, built initially in the special system (3.10), may now be defined as:

$$\tilde{E}_{\lambda\kappa} = \mathrm{i}\tilde{E}_1\gamma^1 + \mathrm{i}\tilde{E}_2\gamma^2 + \tilde{E}_3\gamma^3 = \begin{bmatrix} 0 & \tilde{E}_3 & -\tilde{E}_2 & -\tilde{E}_1 \\ -\tilde{E}_3 & 0 & \tilde{E}_1 & -\tilde{E}_2 \\ \tilde{E}_2 & -\tilde{E}_1 & 0 & -\tilde{E}_3 \\ \tilde{E}_1 & \tilde{E}_2 & \tilde{E}_3 & 0 \end{bmatrix} \quad (3.11)$$

where the values $\{\tilde{E}_s\}$ are treated as the scalars found in the off-diagonal form (3.10); the Dirac tensors of the antisymmetric type, as used here, are given as follows:

$$\gamma^1 = \mathrm{i}\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^2 = \mathrm{i}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \gamma^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

We have chosen the antisymmetric Dirac tensors to enable a comparison with the rotation field ω. Using these definitions we can write for the antisymmetric tensor $\tilde{E}_{\lambda\kappa}$ the relations equivalent to eq. 3.7 (Teisseyre, 2009):

$$\mu\Delta\tilde{E}_{\lambda\kappa} - \rho\frac{\partial^2}{\partial t^2}\tilde{E}_{\lambda\kappa} = Y_{\lambda\kappa} \quad (3.12)$$

where according to (3.7) we will have

$$Y_{\lambda\kappa} = \mathrm{i}Y_{23}\gamma^1 + \mathrm{i}Y_{31}\gamma^2 + Y_{12}\gamma^3 = \begin{bmatrix} 0 & Y_{12} & -Y_{31} & -Y_{23} \\ -Y_{12} & 0 & Y_{23} & -Y_{31} \\ Y_{31} & -Y_{23} & 0 & -Y_{12} \\ Y_{23} & Y_{31} & Y_{12} & 0 \end{bmatrix}$$

and

$$Y_{lq} = -(\lambda+\mu)\left(\frac{\partial^2 E_{ss}}{\partial x_l \partial x_q} - \frac{\delta_{lq}\partial^2 E_{ss}}{3\partial x_k \partial x_k}\right) + \frac{1}{2}\left(\frac{\partial F_n}{\partial x_l} + \frac{\partial F_l}{\partial x_q}\right) - \left(\frac{\partial^2 p}{\partial x_l \partial x_q}\right)$$

Note that there remains an influence of the axial stresses on the deviatoric field. The shear-twist, $\tilde{E}_s$, means the off-diagonal oscillation of shear axes and its amplitude as caused by internal processes. In the special coordinate system, in which we have simplified the deviatoric strains to the off-diagonal form, $\tilde{E}_s$, we have now defined the shear-twist invariant vector form.

The rotation and twist motions form the complex rotation tensor; the related relations joining these fields follow from the standard conservation law in 4D:

$$\tilde{\omega}_{\lambda\kappa} = \omega_{\lambda\kappa} + \mathrm{i}\tilde{E}_{\lambda\kappa}; \quad \frac{\partial}{\partial x^\kappa}\tilde{\omega}_{\lambda\kappa} = \frac{4\pi}{V}J_\lambda; \quad x^\lambda = \{x^1, x^2, x^3, x^4\},\ x^4 = \mathrm{i}Vt \quad (3.13)$$

or

$$\frac{\partial}{\partial x^\kappa}\omega_{\lambda\kappa} = \frac{4\pi}{V}J_\lambda; \quad \frac{\partial}{\partial x^\kappa}\tilde{E}_{\lambda\kappa} = 0;\ x^\lambda = \{x^1, x^2, x^3, x^4\},\ x^4 = \mathrm{i}Vt \tag{3.13a}$$

where we introduced the defect-related current field, J_k, and velocity, V, under the condition that this velocity will be transformed according to the relativistic rules for a sum of velocities.

This system of equations can be split into the twist and rotation Maxwell-like equations:

$$\mathrm{rot}\ \omega - \frac{\mathrm{d}\tilde{\omega}}{V\mathrm{d}t} = 4\pi \mathrm{J}; \quad \mathrm{rot}\ \tilde{\omega} + \frac{\mathrm{d}\omega}{V\mathrm{d}t} = 0 \tag{3.14}$$

where the related velocity depends on the interaction between the rotations and the shear-twist pseudo-vector oscillations of the compression-dilatation axes (or the shear axes shifted by π/4). Note that both fields, rotation and shear, have the azimuth dependent amplitudes.

For the wave equations we obtain:

$$\Delta\omega - \frac{\partial^2}{V^2\partial t^2}\omega = -\frac{4\pi}{V}\in_{npq}\frac{\partial}{\partial x_p}J_q; \quad \Delta\tilde{\omega} - \frac{\partial^2}{V^2\partial t^2}\tilde{\omega} = \frac{4\pi}{V^2}\dot{J}_n + 4\pi\frac{\partial}{\partial x_n}\rho \tag{3.15}$$

where $\tilde{\omega}_s \equiv \tilde{E}_s$ and ω_s present the shear-twist and rotation vectors, respectively, the current relates to defect flow, e.g., dislocations, and ρ relates to defect density.

The idea that the rotation-related amplitudes may differ from the P or S waves arises after experimental study on the velocity of rotation waves (K.P. Teisseyre, private communication, 2009). The relations (3.14) indicate that the rotation wave velocity, V_0, appears as an effect of the mutual interaction between the rotations and shear-twist rotational oscillations.

After Teisseyre et al. (2008) we may write the local solution of the system (3.14) for the twist and spin waves shifted mutually in phases:

$$\omega_s = \pm\mathrm{i}\tilde{\omega}_s, \quad \omega_s = \omega_s^0 exp[\mathrm{i}(k_i x_i - \bar{\omega}t), \quad \tilde{\omega}_s = \tilde{\omega}_s^0 exp[\mathrm{i}(k_i x_i - \bar{\omega}t)] \tag{3.16}$$

where $\omega_s^0 = \pm\mathrm{i}\tilde{\omega}_s^0$.

The related waves, ω_s and $\tilde{\omega}_s$ help us to explain the synchronization of the micro-fracture phenomena; these conjunct solutions show that one of these motions will be delayed in phase by $\pi/2$. Figure 3.1 gives an example of such a synchronization (K.P. Teisseyre, 2007).

Finally, let us note that when comparing our theoretical approach with the experimental measurements obtained, e.g., from the strain-meter or rotation-seismograph systems (strain determination on one plane requires a set of 3 instruments), we should transform these experimental data to the off-diagonal shear values.

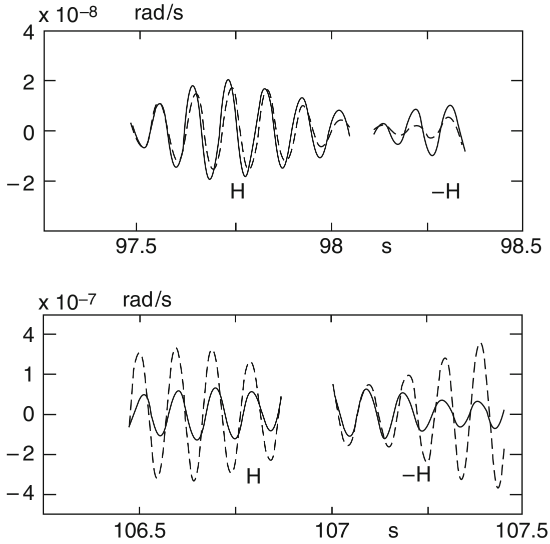

Fig. 3.1 An example of synchronization between the rotations and shear-twist oscillations obtained by applying the Hilbert transform (H)

3.4 Dislocations and disclinations: fragmentation and cracks

In our former papers (Teisseyre 2001, Teisseyre, 2008, Teisseyre and Boratyński 2003) we have introduced the definition of the twist-bend tensor, χ_{mq}:

$$\chi_{mq} = \varepsilon_{ksq} \frac{\partial \omega_{mk}}{\partial x_s} \tag{3.17}$$

which differs from that introduced by Kossecka and DeWitt (1977); according to their definition, the Burgers and Frank vectors would vanish when defining the defects from the twist-bend tensor.

Our definitions, describing the dislocation nuclei, help to obtain the Burgers and Frank vectors and dislocation and disclination densities directly from (3.17):

$$B_l = \oint [E_{kl} + \omega_{kl}] \mathrm{d}l_k \text{ and } \Omega_q = \oint \chi_{pq} \mathrm{d}l_p = \int\int \theta_{pq} \mathrm{d}s_p \tag{3.18}$$

and with the definition

$$B_l = \int\int \left(\alpha_{pl} - \frac{1}{2} \delta_{pl} \alpha_{ss} \right) \mathrm{d}\, s_p \tag{3.19}$$

we obtain the expressions for the defect densities (cf., eq. 3.2):

$$\alpha_{pl} - \frac{1}{2}\delta_{pl}\alpha_{ss} = \varepsilon_{pmk}\frac{\partial}{\partial x_m}\left(E_{kl} + \omega_{kl}\right) = \varepsilon_{pmk}\frac{\partial}{\partial x_m}\left(e^0 E^0_{kl} + \chi^0\,\omega^0_{kl}\right),\ \theta_{pq} = 0 \tag{3.20}$$

and relation with stresses (Teisseyre, 2008),

$$\alpha_{pl} - \frac{1}{2}\delta_{pl}\alpha_{ss} = \frac{\varepsilon_{pmk}}{2\mu}\frac{\partial}{\partial x_m}\left(S_{(kl)} - \frac{v}{1+v}\delta_{kl}S_{ii} + S_{[kl]}\right) \tag{3.21}$$

Another definition of the defect nuclei for the twist-bend tensor can introduce the vortex defects with the specific dislocations and disclinations; when defining:

$$\chi_{mq} = \frac{1}{l}\,\omega_{mq} \tag{3.22}$$

we obtain the same expression for dislocation field, but different for disclinations (cf., eq. 3.20):

$$\theta_{pq} = \varepsilon_{pmk}\frac{\partial \chi_{kq}}{\partial x_m} = \frac{1}{l}\varepsilon_{pmk}\frac{\partial \omega_{kq}}{\partial x_m} = \frac{1}{l}\varepsilon_{pmk}\varepsilon_{kqs}\frac{\partial \omega_s}{\partial x_m} = -\frac{1}{l}\frac{\partial \omega_p}{\partial x_q} \tag{3.23}$$

Disclinations related to gradient of rotation become the vortex-defects. An array of the vortex-defects can help us to approximate the fragmentation/cracks (similarly as an array of dislocations approximates a crack).

Finally, we obtain the relation for disclinations and antisymmetric stresses as follows

$$\theta_{pq} = \frac{1}{l}\varepsilon_{pmk}\frac{\partial \omega_{kq}}{\partial x_m} = \frac{1}{2\mu}\varepsilon_{pmk}\frac{\partial S_{[kq]}}{\partial x_m} \tag{3.24}$$

3.5 Interaction fields

First, we recall that the two independent fields, E_{lk} and ω_{kl}, or equivalently, $\tilde{E}_{\lambda\kappa}$ and $\omega_{\lambda\kappa}$, subjected together to the equations of motions (eqs. 3.6, 3.7, 3.9 or 3.13), can be directly coupled by the phase-delayed solution as written in special off-diagonal coordinate system (3.16):

$$E^D_{kl} = \pm \mathrm{i}\omega_{kl} \tag{3.25}$$

Presenting the theory of interaction processes we can write a very general form of the constitutive laws (Teisseyre, 2008):

$$S_{(kl)} = 2\mu\left(e^0 E^0_{kl} + e'\delta_{kl}H + e''H_{(kl)}\right), \quad S_{[kl]} = 2\mu\left[\chi^0\omega^0_{kl} + \chi'\varepsilon_{kls}G_s + \chi''G_{[kl]}\right] \tag{3.26}$$

where $E^0_{kl} = \left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l}\right)$, $\omega^0_{kl} = \left[\frac{\partial u_l}{\partial x_k} - \frac{\partial u_k}{\partial x_l}\right]$ and H, $H_{(kl)}$, G_s, $G_{[kl]}$ are the non-mechanical stress-influencing fields; the constants we introduced, e^0, e', e'' and χ^0, χ', χ'', are the phase constants which may vary from 0 to $\pm$ 1or $\pm$ i. According to standard asymmetric theory, we relate the strain and rotation with displacements according to eq. 3.2.

However, considering the specific cases separately we can assume that an influence of the mechanical fields, E^0_{kl} or ω^0_{kl}, on the other physical fields (e.g., electric ones) is direct; therefore, it will be enough to assume that the phase shift constants are equal: $e^0 = e' = e''$ and $\chi^0 = \chi' = \chi''$. This assumption means that the interaction between the deformations and non-mechanical fields proceeds without a delay (no phase shift), while the coupling between the mechanical fields themselves may occur with the phase delay, as given in relation (3.25) describing the release-rebound process.

Therefore, further on, instead of (3.26), we write:

$$\begin{aligned} S_{(kl)} &= 2\mu E_{kl} = 2\mu e^0\left(E^0_{kl} + \delta_{kl}H + H_{(kl)}\right), \\ S_{[kl]} &= 2\mu\omega_{kl} = 2\mu\chi^0\left(\omega^0_{kl} + \varepsilon_{kls}G_s + G_{[kl]}\right) \end{aligned} \tag{3.27}$$

Thus, in our approach the elastic deformation fields can be defined as follows:

$$E_{kl} = e^0\left(E^0_{kl} + \delta_{kl}H + H_{(kl)}\right), \quad \omega_{kl} = \chi^0\left[\omega^0_{kl} + \varepsilon_{kls}G_s + G_{[kl]}\right] \tag{3.28}$$

The symmetric and antysimmetric stresses remain to be given, in an elastic regime, by relations (3.4).

We should keep in mind that, in the Kröner metod, the physically significant elastic fields, S_{ks}, E_{ks}, ω_{ks}, are given by the differences between the total fields, S^0_{ks}, E^0_{ks}, ω^0_{ks} (related directly to the displacement differentials), and the self fields, S^S_{ks}, E^S_{ks}, ω^S_{ks} (related to internal interaction nuclei): $S_{ks} = S^0_{ks} - S^S_{ks}$, $E_{ks} = E^0_{ks} - E^S_{ks}$, $\omega_{ks} = \omega^0_{ks} - \omega^S_{ks}$.

It is only the total field that preserves the usual symmetry properties: elastic and self fields may be asymmetric. A comparison of our approach and that used used in the Kröner method was given by Teissyere (2008); we recall here only that the interaction fields in the Kröner theory enter through the self-nuclei whose fields appear in the self- stress, self-strain and self-rotation fields; the relation between the total, elastic and self fields is the following:

$$S^T_{ks} = S^E_{ks} + S^S_{ks}, \quad E^T_{ks} = E^E_{ks} + E^S_{ks}, \quad \omega^T_{ks} = \omega^E_{ks} + \omega^S_{ks}$$

In the Kröner theory, the elastic fields represent the physical fields; the total field preserves the usual symmetry properties, while the elastic and self fields may be asymmetric.

In our approach the stresses are asymmetric, as explained at the beginning.

3.6 Direct relations between defect and electric fields

Returning to the derived relations (3.21) and (3.24) we rewrite them, by virtue of (3.27), as:

$$
\begin{aligned}
&e^0 \varepsilon_{pmk} \frac{\partial}{\partial x_m} \left(\left(E_{kl}^0 + \delta_{kl} H + H_{(kl)} \right) - \frac{v}{1+v} \delta_{kl} e^0 \left(E_{ii}^0 + 3H + H_{(ii)} \right) \right) + \\
&\chi^0 \varepsilon_{pmk} \frac{\partial}{\partial x_m} \left(\omega_{kl}^0 + \varepsilon_{kls} G_s + G_{[kl]} \right) = \alpha_{pl} - \frac{1}{2} \delta_{pl} \alpha_{ss}
\end{aligned}
\tag{3.29}
$$

and

$$
\chi^0 \varepsilon_{pmk} \frac{\partial}{\partial x_m} \left(\omega_{kl}^0 + \varepsilon_{kls} G_s + G_{[kl]} \right) = \theta_{pq} \tag{3.30}
$$

These relations could be used as the differential equations for a chosen non-mechanical field (selected from the set: $H, H_{(kl)},\ G_s,\ G_{[kl]}$) to estimate directly its influence on the defect fields (given dislocation and disclination densities); or to find an influence of defects on the non-mechanical fields.

3.7 Interaction examples

3.7.1 Thermal interaction

For a thermal field, we write a more generalized relation than that in the classic elastic theory:

$$
S_{(kl)} = 2\mu e^0 \left(E_{kl}^0 - \delta_{kl} \alpha^{\mathrm{th}} (T - T_0) \right), \quad S_{[kl]} = 2\mu \chi^0 \omega_{kl}^0 \tag{3.31a}
$$

Comparing with (3.27) we put

$$
S_{(kl)} = 2\mu e^0 \left(E_{kl}^0 + \delta_{kl} H \right), \qquad S_{[kl]} = 2\mu \chi^0 \omega_{kl}^0 \tag{3.31b}
$$

where $H = -\alpha^{\mathrm{th}} (T - T_0)$ and where for $e^0 = 1$ and $\chi^0 = 0$ we return to the classic case.

The equivalent relation between this thermal field and the dislocations becomes:

$$
\varepsilon_{pml} \frac{\partial}{\partial x_m} \left(E_{kl}^0 - e^0 \alpha^{\mathrm{th}} \frac{1-2v}{1+v} (T - T_0) \right) = \alpha_{pl}^{\mathrm{edge}} \tag{3.32}
$$

and there is no contribution from screw dislocations.

3.7.2 Piezoelectric effects

The classical piezoelectric effect appears in anisotropic crystals, piezoelectric dielectrics; after Toupin (1956; see: Mindlin, 1972, Teisseyre, 2001a) we write the constitutive law as:

$$S_{ij} = 2\mu E_{kl} - e_{kij} E_k \tag{3.33a}$$

where E_k is the electric field, e_{kij} are the piezoelectric stress constants.

We can rewrite this relation as follows:

$$S_{(ij)} = \lambda \delta_{ij} E_{ss} + 2\mu E_{kl} - e_{k(ij)} E_k, \; S_{[ij]} = 2\mu \omega_{kl} - e_{k[ij]} E_k \tag{3.33b}$$

According to our approach (3.28) we obtain:

$$E_{kl} = e^0 \left(E^0_{kl} + h_s \delta_{kl} E_s + \varepsilon_{s(kl)} g E_s \right), \quad \omega_{kl} = \chi^0 \left(\omega^0_{kl} + \varepsilon_{s[kl]} g E_s \right) \tag{3.33c}$$

where we have separated the piezoelectric constant into its symmetric and anti-symmetric parts and introduced other definitions:

$$e_{kij} = -2\mu \left(h_k \delta_{ij} + \varepsilon_{kij} g \right) \quad \text{and} \quad H\delta_{ij} = h_k \delta_{ij} E_k, \; G_{[ij]} = \varepsilon_{kij} g E_k \tag{3.34}$$

The equivalent relation between this piezoelectric field and the defect densities becomes:

$$e^0 \varepsilon_{pmk} \frac{\partial}{\partial x_m} \left(\left(E^0_{kl} + h_s \delta_{kl} E_s \right) - \frac{v}{1+v} \delta_{kl} \left(E^0_{ss} + 3 h_s E_s \right) \right) + \chi^0 \varepsilon_{pmk} \frac{\partial}{\partial x_m} \left(\omega^0_{kl} + \varepsilon_{skl} g E_s \right)$$
$$= \alpha_{pl} - \frac{1}{2} \delta_{pl} \alpha_{ss} \tag{3.35a}$$

$$\varepsilon_{pmk} \frac{\partial \chi_{kq}}{\partial x_m} = \frac{1}{l} \chi^0 \varepsilon_{pmk} \frac{\partial}{\partial x_m} \left(\omega^0_{kq} + \varepsilon_{skq} g E_s \right) = \theta_{pq} \tag{3.35b}$$

We note that the piezoelectric constants for various crystallographic classes have been discussed by Nowacki (1983).

3.7.3 Polarization gradient theory

According to Mindlin (1972), the internal energy depends also on the polarization gradient; we can write:

$$\Pi_{ij} = \frac{\partial \Pi_i}{\partial x_j} \tag{3.36}$$

where polarization, $\Pi_i = D_i - \varepsilon E_i$, is defined by difference of electric displacement, D, and electric field, E, with ε being the permittivity of vacuum. The gradient theory, related to electric polarization, makes use of the fact that, under the applied load, the displacements of a moving dislocation core (electrically charged) influence the surrounding defect cloud (such a cloud shall have the opposite charge, compensating that of a dislocation core).

The constitutive relations (Mindlin, 1972; Nowacki,1983) with the respective material constants can be written as follows:

$$S_{ij} = 2\mu E_{ij} + f_{kij}\Pi_k + d_{klij}\Pi_{kl} \tag{3.37}$$

and, according to relations (1-3), can be generalized for the asymmetric stresses to the following form (cf., Teisseyre, 2001):

$$S_{(ij)} = \lambda\delta_{ij}E_{ss} + 2\mu E_{ij} + f_{k(ij)}\Pi_k + d_{kl(ij)}\Pi_{kl}, \quad S_{[ij]} = 2\mu\omega_{ij} + f_{k[ij]}\Pi_k + d_{kl[ij]}\Pi_{kl} \tag{3.38}$$

Now, we can present the contributions to the asymmetric strains and rotations caused by the electric polarization coupling:

$$E_{ij} = e^0\left(E_{ij}^0 + \frac{1}{2\mu}f_{k(ij)}\Pi_k + \frac{1}{2\mu}d_{kl(ij)}\Pi_{kl}\right) \tag{3.39a}$$

$$\omega_{ij} = \chi^0\left(\omega_{ij}^0 + \frac{1}{2\mu}f_{k[ij]}\Pi_k + \frac{1}{2\mu}d_{kl[ij]}\Pi_{kl}\right) \tag{3.39b}$$

For the direct relation with defects we write according to (3.29) and (3.30):

$$\begin{aligned} e^0\varepsilon_{pmk}\frac{\partial}{\partial x_m}&\left(\left(E_{kl}^0 + \frac{1}{2\mu}f_{k(ij)}\Pi_k + \frac{1}{2\mu}d_{kl(ij)}\Pi_{kl}\right)\right. \\ &\left.- \frac{v}{1+v}\delta_{kl}e^0\left(\frac{1}{2\mu}f_{k(ss)}\Pi_k + \frac{1}{2\mu}d_{kl(ss)}\Pi_{kl}\right)\right) \\ &+ \chi^0\varepsilon_{pmk}\frac{\partial}{\partial x_m}\left(\omega_{kl}^0 + \varepsilon_{kls}G_s + G_{[kl]}\right) = \alpha_{pl} - \frac{1}{2}\delta_{pl}\alpha_{ss} \end{aligned} \tag{3.40a}$$

$$\chi^0\varepsilon_{pmk}\frac{\partial}{\partial x_m}\left(\omega_{kl}^0 + \varepsilon_{kls}G_s + G_{[kl]}\right) = \theta_{pq} \tag{3.40b}$$

Moreover, note that some experiments (see: e.g., Hadijcondis and Mavromatou, 1994, 1995) indicate that the anomalous piezoelectric effects, observed in the laboratory experiments, correspond to the time rate of the applied load.

The problem of magnetostrictive effects can be treated in a similar way.

3.7.4 Interaction chains: electric and acoustic effects

Finally, we can note that the shear and axial stresses influence (cf., eq. 3.7) the solution for the deviatoric stresses, E^D_{nl}, and, further on, these strains can influence the rotation field (cf., eq. 3.14); we can express this coupling also by one of possible solutions of the system (3.16):

$$\omega_{\lambda\kappa} = \pm \mathrm{i}\tilde{E}_{\lambda\kappa} \tag{3.41}$$

It seems reasonable to believe that the coupling between the mechanical and electric (or electric polarization) field proceeds in an instantaneous manner, because such effects follow from the displacement of the ions. However, as shown in (3.29), the interaction.between the mechanical fields can proceed with a phase delay due to the release-rebound sequence. Hence, we can have different interaction chains (cf., eqs 3.7 and 3.12) like, e.g., the following ones:

$$E^D_{nl} \rightarrow \mathrm{i}\omega_{nl} \rightarrow \mathrm{i}\Pi_s \tag{3.42a}$$

where the shears coupled to the phase-delayed rotations lead to polarization effects,

$$p \rightarrow E^D_{nl} \rightarrow \mathrm{i}\omega_{nl} \rightarrow \mathrm{i}\Pi_s \tag{3.42b}$$

where a pressure variation (mechanical forcing) initiates a similar chain,

$$E_s \rightarrow \omega_{nl} \rightarrow \mathrm{i}E_{nl} \tag{3.42c}$$

where the electric field variations force rotation effects and the micro-strain releases revealed by the acoustic bursts occurring with the phase delay.

3.8 Conclusions

– We have presented the asymmetric continuum theory including different types of material states: from elastic continuum to granulated/crushed material.
– We have assumed the balance relation for the antisymmetric stresses as equivalent to that for the stress couple. We have defined the 4D invariant form of the shear-twist field.
– The spin and the shear-twist oscillation of the off-diagonal shear axes led us to the relations for the rotation and rotational shear-twist waves.
– We have presented a new definition for dislocation and disclination density field permitting to derive the relations between the asymmetric stresses and linear defect densities.

- We have presented a new relation for the interaction between the strains and rotations and other physical fields; these relations are more general than those between the stresses and some physical fields as, in this new approach, we consider the asymmetric fields and also we may include a phase shift when a rebound process provoked by some energy release shall be considered.
- The direct relations are given between the defect densities and the non-mechanical fields.
- Some examples are given for the interaction between the strains or rotations on the one side, and the electric and electric polarization fields on the other.
- The experimental evidence for the appearance of spin and twist motion in a seismic field is based on the records of the seismic rotation fields (see: Teisseyre at al., 2006; Teisseyre K.P, 2007). Comparison between the experimental data (e.g., strain variation in time as can be obtained from the strain-meter or rotation-seismograph systems) and theoretical consideration on twist field (shears in the off-diagonal system) require transformation of the obtained theoretical twist motion values to the diagonal shear ones.
- The asymmetric continuum theory includes description of the states close to micro-fracture processes; the hypothesis on the local synchronization, related to the special complex solution for the rotation and twist fields, is confirmed by some correlations observed between the recorded twist and spin seismic wave groups.

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