Chapter 2
Models of Stick-Slip Motion: Impact of Periodic Forcing

T. Chelidze and N. Varamashvili

Abstract The modern concept of seismic processes relays mainly on the model of frictional instability, which develops on the preexisting tectonic fault, in contrast to the earlier assumptions on the brittle fracture of the crust material attaining the critical stress.

The Ditrich-Ruina equation for shear stress describes almost all main features of slip, obtained in numerous experiments: it shows that the frictional force is not a constant, but is time-dependent and undergoes complex evolution during the slip event. The equation is nonlinear, and consequently the slip process should manifest such properties as high sensitivity to weak external forcing, hysteresis effect, etc. It is quite natural that the instabilities of friction excite vibrations, including acoustic emission (AE). The AE is expected to occur during slips and be absent during stick phase. We presume that acoustic measurements may reveal the fine details of friction mechanism, which are beyond the reach of direct displacement-measuring techniques.

The additional forcing, which can be much smaller than the main driving force, may provoke triggering and synchronization during stick-slip process, which means that these phenomena are invoked by nonlinear interaction of objects. An attempt to compile and analyze the rate- and state slip equation taking into account the periodic forcing is made.

2.1 Introduction

It is well known from the surface physics that the friction (adhesion) force *Ff* is a result of intermolecular and intersurface forces of mainly electromagnetic origin: (i) purely electrostatic (Coulomb) forces, (ii) polarization due to the induced dipole moments, and (iii) quantum-mechanical forces. Friction results in transmission and

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Table 2.1. A capsule story of friction models:

Time Arrow ↓	Amonton, 1699	$\tau = \sigma_n \mu$
	Coulomb, 1773	$\tau = c + \sigma_n \mu$
	Hubbert and Rubbey, 1959	$\tau = c + \mu(\sigma_n - P_p) = c + \sigma\mu_{eff}$
	Brace and Byerlee, 1966	$\tau = \sigma_0(\mu_0 + a\ ln(V/V_o) + bln(V\Theta/D_0))$
	Burridge-Knopoff, 1967	$d\Theta/dt = 1 - (V\Theta/D_0)$
	Dietrich, 1972,	
	Ruina, 1983	

dissipation of energy. Kinetic energy of motion is converted into thermal energy mostly by acoustic processes. Instability in sliding occurs when the friction pumps to the system more energy than can be dissipated by the stationary process.

The modern concept of seismic process relays mainly on the model of frictional instability, which develops on the preexisting tectonic fault, in contrast to the earlier assumptions on the brittle fracture of the crust material at attaining the critical stress. The first simple friction models suggested by Amonton and Coulomb were refined by Hubert and Rubbey (1979), Brace and Byerlee (1966), Burridge and Knopoff (1967), Dieterich (1979) and Ruina (1983): the capsule story of friction models, showing main stages of development in this area, is presented in Table 2.1.

Here τ and σ_n are shear and normal stresses, respectively, μ is the friction coefficient, c is the adhesion term, P_p is the pore pressure, V and V_0 are current and initial velocities of drag, Θ is the state variable, D_0 is the critical slip distance, and a and b are constants.

The last expression for shear stress describes almost all main features of slip, obtained in numerous experiments: it shows that the frictional force is not a constant, but is time-dependent and undergoes complex evolution during slip event. The equation is nonlinear, and consequently the slip process should manifest such properties as high sensitivity to weak external forcing, hysteresis effect, etc.

2.2 Main details of experimental stick-slip results

Depending on conditions (spring stiffness k, velocity of drag V, normal stress σ_n, slip surface state θ), three main types of friction are observed by displacement recording: stick-slip, inertial regime, and stable regime. Figure 2.1 shows spring deflection δx, top plate position x and its instantaneous velocity V during stick-slip motion.

The stick-slip regime is observed at relatively low velocities V and low stiffness. At higher V, the transition to inertial periodic oscillations occurs; at still higher V we have the stable sliding with fluctuations.

The single slip events were investigated in detail by Nasuno et al (1997): after application of tangential force, the velocity of slip drastically increases and then decreases (Fig. 2.2).

The instantaneous frictional force $\mu(t) = F_f/Mg$ during the slip event experiences hysteresis (Fig. 2.3): during the stick stage, μ increases until the static

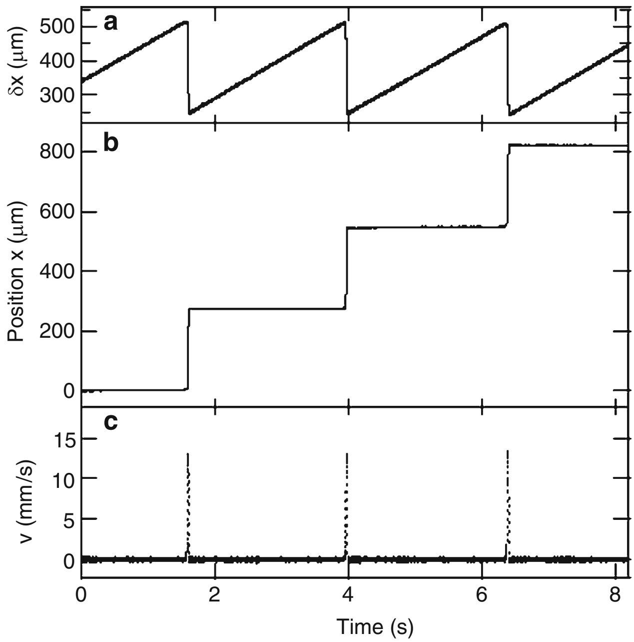

Fig. 2.1 Spring deflection δx top plate position x and its instantaneous velocity V during stick-slip motion (Nasuno et al. 1997)

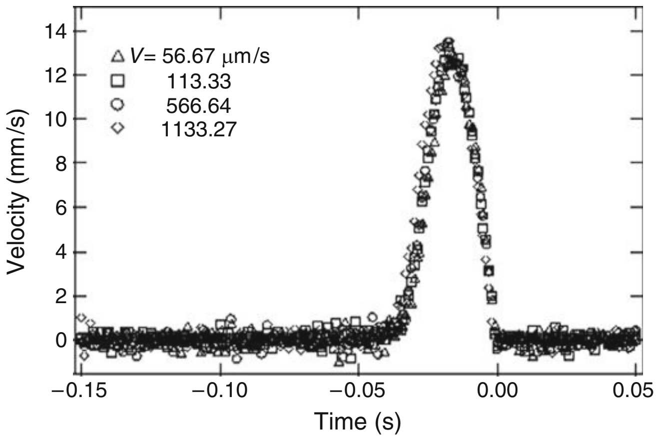

Fig. 2.2 The slip velocity evolution during single slip event for various driving velocities V = 56.67; 113.33; 566.64 and 1133.27 mm/s (Nasuno et al. 1997)

threshold $\mu_s = F_s/Mg$ is attained, and the slip begins. During slip, μ decreases to its kinetic value; after this, at the deceleration stage it drops to the initial value μ_0 (Nasuno et al. 1998).

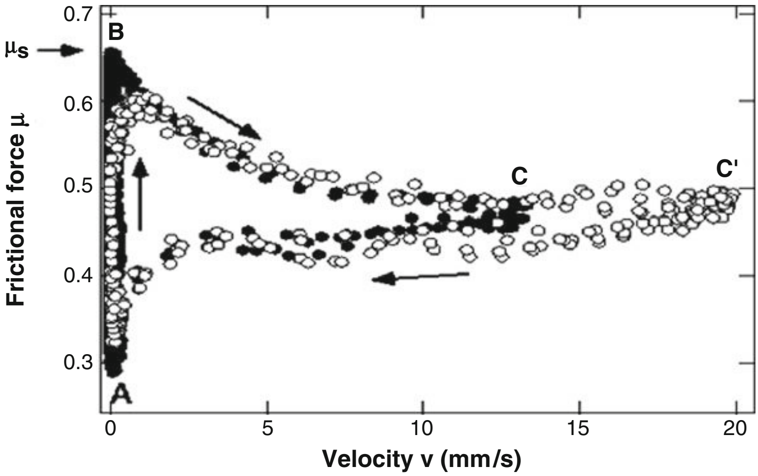

Fig. 2.3 Frictional force μ versus slip velocity during a slip event (Nasuno et al. 1998)

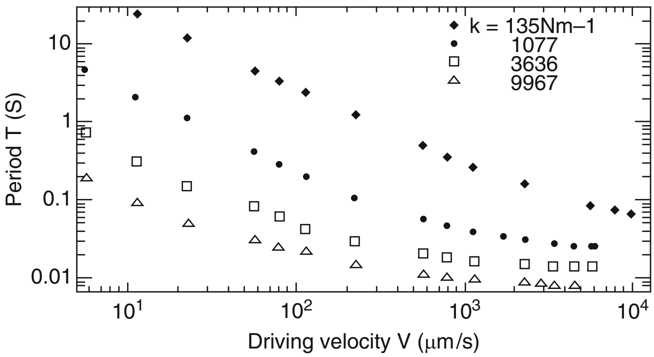

Fig. 2.4 Dependence of slip recurrence period T on velocity V and stiffness k (Nasuno et al. 1998)

Mean period of stick-slip motion T depends on the drive velocity V and spring stiffness k; $T \sim 1/V$ at low V and T decreases with increase of k (Fig. 2.4; Nasuno et al. 1998).

For understanding the physics of stick-slip motion it is very interesting to note that each slip is connected with relatively slow vertical displacement of D_v of the (top) sliding plate relative to the fixed lower plate; it is evident that the maximum of D_v precedes the maximum of tangential velocity V_t. This means that before the slip in horizontal direction, the top plate is rising up; evidently, the plate is ascending and the large asperities, which prevent slip and the slip displacement, occur at reaching the critical number of contact points (threshold). This suggestion is confirmed by the above-mentioned experimental evidence of small vertical displacement preceding the slip event (Fig. 2.5), which means that the number of contact points n decreases to some threshold value n_c making possible the

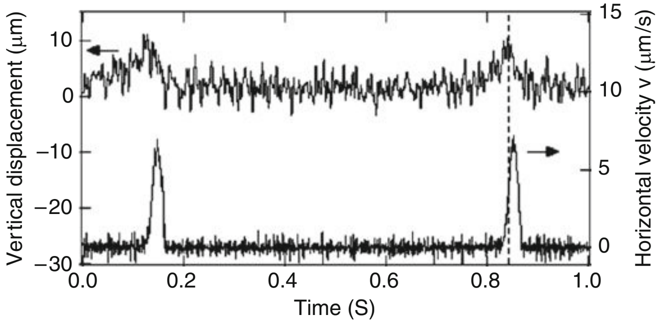

Fig. 2.5 Vertical displacement D_v and tangential velocity V_t of the top plate versus time (Nasuno et al. 1998)

macroscopic tangential displacement. The mathematical formalism, similar to that of percolation model of fracture, could be developed for the slip process (Chelidze, 1986). It seems that the percolation theory, namely, the model of percolation for tangential shift of contacting fractal surfaces, may explain the transition of friction coefficient from the static to kineticvalue at attaining some critical value of contact points of shearing fractal surfaces. In Chelidze (1986) the guess is given about a possibility of applying the percolation model of fracture to tectonic fault dynamics.

It is quite natural that the instabilities of friction excite vibrations, including acoustic emission (AE). The reverse effects are also observed, namely, vibrations affect the friction (Akay, 2002; Chelidze, Varamashvili et al., 2002; Chelidze and Lursmanashvili, 2003; Chelidze, Gvelesiani et al., 2004; Chelidze, Matcharashvili et al., 2005; Chelidze and Matcharashvili, 2007; Chelidze, Lursmanashvili et al., 2006). We presume that acoustic measurements may reveal the fine details of friction mechanism, which are beyond the reach of direct displacement-measuring techniques. The situation is similar to brittle fracture studies, where AE is much more sensitive to micro-fracturing than traditional stress-strain experiments.

In this connection, we presume that the so-called stable sliding is not stable at all, but involves fast micro-events that can not be registered by (slow) displacement sensors.

2.3 Mathematical models of friction

The mathematical expressions for the shear stress τ, formulated by Dietrich and Ruina (Table 2.1) are in agreement with the majority of observed data on stick-slip. It is shown that for some critical stiffness k_c the system undergoes Hopf bifurcation, leading finally to instability. The solution of the system in this case demonstrates all details, characteristic for (chaotic) nonlinear dynamics (Becker, 2000).

An analysis of the experimental data obtained by investigating of spring-slider system motion has led to empirical law, named rate- and state-dependent friction law (Dieterich, 1979; Ruina, 1983). When the sliding velocity is changed in laboratory friction experiments, two effects are seen to occur to the dynamic coefficient of friction (Bureau et al, 2000; Kanamori and Brodsky, 2004; Boettcher and Marone, 2004). First, there is a “direct” effect that opposes the change in velocity. Hence, if the velocity is increased, the dynamic friction coefficient will correspondingly rise (Fig. 2.2). If the sliding velocity is reduced, the dynamic friction coefficient will drop. This can be described as “rate-dependent friction”. The second effect refers to the fact that, after abrupt changes in velocity, the frictional resistance evolves to a new steady state over a characteristic slip distance D_0; this is termed “evolution effect”.

The rate and state dependent friction can be quantified as follows (Dieterich, 1979; Ruina, 1983; Kanamori and Brodsky, 2004; Scholz, 1998).

$$\tau = \sigma_o\left(\mu_o + a\ ln\left(\frac{V}{V_o}\right) + b\ ln\left(\frac{V_o\theta}{D_o}\right)\right), \tag{2.1}$$

where μ_0 is the initial coefficient of friction, V is the new sliding velocity, V_0 is the initial sliding velocity, θ is the state variable, D_0 is the critical slip distance, and a and b are two experimentally determined constants.

The state variable varies according to:

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_o} \tag{2.2}$$

In the spring-slider model, dependence of upper sliding plate velocity on time can be graphically presented as shown in Fig. 2.6:

For qualitative analysis of processes, in the transient area between stages 1 and 2 (near stage 2), the equation of motion for this system, under the assumption of $\left|\frac{\theta V}{D_0}\right| \gg 1$, can be written as (Kanamori and Brodsky, 2004):

$$\sigma_o\left(\mu_o + a\ ln\ \dot{x} + b\ ln\ \theta_o - \frac{b}{D_o}x\right) = -kx + kx_o, \tag{2.3}$$

where $\dot{x}$ represents displacement, x_0 the spring elongation, and k the spring stiffness.

By integration of (2.3) for the initial conditions $x = 0$ and $\dot{x} = \dot{x}_0$ for t = 0, we obtain:

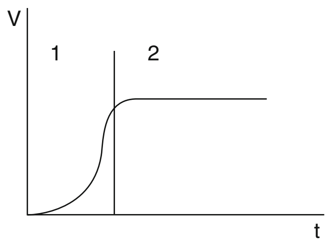

Fig. 2.6 Velocity vs. time during a stick-slip motion

$$\dot{x} = \left[\frac{1}{\dot{x}_o} - \frac{Ht}{a}\right]^{-1} \quad \text{where } H = -\frac{k}{\sigma_o} + \frac{b}{D_o} \tag{2.4}$$

From (2.4) it follows that the sliding velocity spontaneously increases at the time $t_f = \frac{a}{H}(1/\dot{x}_0)$. That is system's destabilizing (relaxation) time. We can say that t_f is a period of stick-slip for our system.

In the case of addition of periodical normal forcing to the main driving force, we can write the equation of motion for our system in the following way (Bureau et al. 2000; Varamashvili and Simonishvili, 2005; Varamashvili, 2006; Putelat et al., 2007):

$$m\frac{d^2x}{dt^2} = k(Vt - X) - W\left(\mu_o + a_o \ln\left(\frac{V}{V_o}\right) + b_o \ln\left(\frac{\theta V_o}{D_o}\right)\right) \tag{2.5}$$

$$\frac{d\theta}{dt} = 1 - \frac{\theta V}{D_o} - \frac{aW}{bW}\theta \tag{2.6}$$

where $W = W_o(1 + \varepsilon\cos(\omega t))$, $W_0\varepsilon$ is the amplitude of forcing, ω is the frequency of forcing, and $T = \frac{2\pi}{\omega}$ is the period of forcing.

In the received system we will solve equation (2.6) to obtain T and we will insert the obtained solution into equation (2.5). For definite parameters from equation (2.5) we obtain the following equation:

$$\ddot{x} + 0.1t\dot{x} + 100t\dot{x} + 100x - 45 - 2\ln\left[1 + \sec\left(\frac{t}{4}\right)^2\right] = 0 \tag{2.7}$$

We solved equation (2.7) using numerical method and the solution is presented in graphic form in Fig. 2.7.

In Fig. 2.7, on the X axis is the current time, and on the Y axis the displacement. From this figure it is evident that for the given parameters the displacement is periodic and decreasing. In fact, experiments show that the stick-slip process has a quasi-periodic character. To simulate the quasi-periodic process we enter periodic normal pressure into equation (2.6) with one-order larger period than the period of

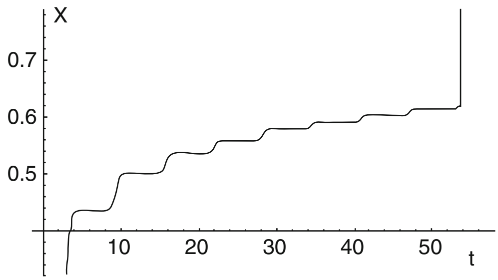

Fig. 2.7. Periodic and decreasing displacement of sliding plate

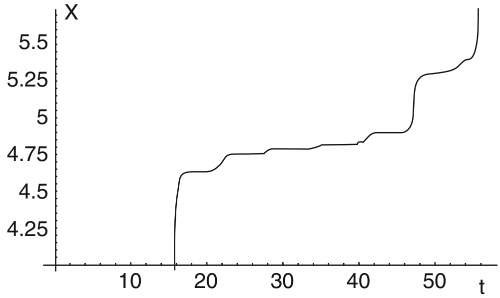

Fig. 2.8 Quasiperiodic displacement of sliding plate according to equation (2.8)

natural stick-slip at corresponding parameters. The idea is to simulate roughness of adjoining surfaces by large period normal loading (we presume that the roughness of surfaces leads to quasi-periodicity of stick-slip process). By means of change of parameters we can try to simulate sliding surfaces of blocks. For definite parameters in equation (2.5) we receive the following equation:

$$\ddot{x} + 6\log(\dot{x}) + 1.7t\dot{x} + 6.8x - 10.9\log\left[\sec\left(\frac{t}{2}\right)^2\right] + 3\cos(0.0999t)\log(\dot{x})$$
$$+ 3\cos(0.0999t)t\dot{x} + 3\cos(0.0999t)\log\left[\sec\left(\frac{t}{2}\right)^2\right] = 0 \qquad (2.8)$$

By solving this equation numerically and presenting the solution graphically, we get Fig. 2.8.

From Fig. 2.8 it is evident that for given parameters the stick-slip process has a quasiperiodic character that reflects experimental data.

For further processing of the method we should try to go from the qualitative agreement of theoretical data with experimental ones to their quantitative conformity.

For solving this system of differential equations (eqs. 2.5 and 2.6), we should make it dimensionless. We introduce dimensionless variables in the following way: dimensionless coordinate is $x = \frac{X}{x_s}$, where x_s is coordinate center of mass of the upper plate in the steady state, dimensionless time is $\tau = \frac{t}{T}$, where T is the period of forcing, dimensionless velocity is $v = \frac{V}{v_s}$, where v_s is the velocity of the steady state, dimensionless state variable is $\vartheta = \frac{\theta}{\theta_s}$, where $\theta = \frac{D_0}{v_s}$ is the state variable at the steady state, characteristic length is $l = v_s T$. After making equations (2.5) and (2.6) dimensionless we obtain:

$$\frac{d^2x}{dt^2} = \beta_1\left(\tau\frac{dx}{d\tau} - x\right) - \beta_2(1 + \varepsilon\cos(2\pi\tau))\left[1 + a\ ln\left(\frac{v}{v_o}\right) + b\ ln\left(\beta_3\frac{\vartheta v_o}{d_o}\right)\right] \qquad (2.9)$$

$$\frac{d\vartheta}{d\tau} = \frac{T}{\theta_s} - \frac{\vartheta v}{d_o} - \frac{a\omega}{b\omega}\vartheta, \qquad (2.10)$$

with the dimensionless parameters

$$v_o = \frac{v_o}{v_s}, \quad a = \frac{a_o}{\mu_o}, \quad b = \frac{b_o}{\mu_o}, \quad d_o = \frac{D_o}{\iota},$$

$$\beta_1 = \frac{kT^2}{m}, \quad \beta_2 = \frac{W_o \mu_o T^2}{mx^5}, \quad \beta_3 = \frac{\theta_s v_s}{\iota},$$

If the forcing amplitude is small as compared to the constant component ($\varepsilon \ll 1$), we can use perturbation theory and write the coordinate, velocity center of mass of the upper plate, and the state variable as:

$$x = 1 + \delta x, \quad v = 1 + \delta v, \quad \vartheta = 1 + \delta\vartheta$$

where δx, δv, $\delta\vartheta$ are small additions.

After simple transformation, equation of the upper plate center mass motion in first order of perturbation theory comes to the equation for harmonic oscillator with variable external force and friction:

$$\delta\ddot{x} + \gamma_1(\tau)\delta\dot{x} + \gamma_2(\tau)\delta x = f(\tau) \tag{2.11}$$

where

$$\gamma_1 = [\beta_1\tau - a\beta_2(1 + \varepsilon\cos(2\pi\tau))],$$

$$\gamma_2 = \frac{b}{d_0}\beta_2(1 + \frac{d_o}{b} + \varepsilon\cos(2\pi\tau)),$$

$$\begin{aligned} f(\tau) = \beta_1 + \beta_2(1 + \varepsilon\cos(2\pi\tau))\Bigg[1 - a\ ln\ v_o + b\ ln\frac{\beta_3 v_o}{d_o} + \frac{b}{d_o^2}e^{\frac{1}{d_o}\tau} \\ \times \int_o^\tau \delta x e^{\frac{1}{d_o}t}dt + \frac{2\pi\varepsilon\alpha}{b}e^{\frac{-1}{d_o}}\int_o^\tau e^{\frac{1}{d_o}t}\frac{\sin(2\pi t)}{1 + \varepsilon\cos(2\pi t)}dt\Bigg] \end{aligned} \tag{2.12}$$

If the variable external forcing is zero, then one of the solutions of homogeneous equation for harmonic oscillators presents Hermitian polynomial.

The general solution of inhomogeneous equation of second order is:

$$\ddot{z} + P(\tau)\dot{z} + Q(\tau)z = F(\tau) \tag{2.13}$$

The general solution for homogeneous equation of harmonic oscillator is:

$$z^0(\tau) = Az_1^0(\tau) + Bz_2^0(\tau)$$

where $Z_1^0(\tau)$ presents Hermitian polynomial, and $Z_2^0(\tau)$ can be expressed through $Z_1^0(\tau)$ using the known relation:

$$z_2^o(\tau) = z_1^o(\tau)W(0)\int_o^\tau \frac{e^{-\int_o^{\tau'} P d\tau''}}{z_1(\tau')^2} d\tau'$$

where W is Wronskian.

The general solution of inhomogeneous equation (2.13) is the sum of partial solution of inhomogeneous equation $z^1(\tau)$ and general solution of homogeneous equation $z^0(\tau)$:

$$z(\tau) = z'(\tau) + z^o(\tau)$$

where

$$z'(\tau) = \int_0^\tau \frac{z_2^0(\tau)z_1^0(\tau') - z_1^0(\tau)z_2^0(\tau')}{W} F(\tau')d\tau'$$

The solution of eq. (2.13) is quite complicated, but it can give new insights in the dynamics of stick-slip motion.

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